DOCUMENT RESUME

ED 175 640 SE 027 881

AUTHOR	Harpel, Jim; And Others
TITLE	CSMP Mathematics for the Intermediate Grades Part II, Teacher's Guide. General Introduction. Workbooks. Experimental Version.
INSTITUTION	Central Midwestern Regional Educational Lab., St. Ann, Mo.
SPONS AGENCY	National Inst. of Education (DHEW), Washington, D.C.s
PUB DATE	77
NOTE	198p.; For related documents, see SE 027 875-893; Not available in hard copy due to copyright restrictions; Contains colored charts and activities which may not reproduce well
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Curriculum Guides; *Elementary School Mathematics; Instructional Materials; Intermediate Grades; *Mathematical Logic; Mathematics Curriculum; *Mathematics Instruction; Multiplication; *Number Concepts; Textbooks; Workbooks
IDENTIFIERS	*Comprehensive School Mathematics Program

ABSTRACT

This guide represents the final experimental version of a pilot project which was conducted in the United States between 1973 and 1976. The ideas and the manner of presentation are based on the works of Georges and Frederique Papy. They are recognized for having introduced colored arrow drawings ("papygrams") and models of our numeration system (the Papy "minicomputer") into the teaching of mathematics at the elementary and secondary level in Belgium. This program follows the "spiral approach." Suggestions are given for obtaining materials for lessons. A Day-by-Day Guide suggests the organization of the material in an 18-week period. Three workbooks are included which provide students with individualized work. Two story-workbooks are also included. The workbooks are written at three levels of difficulty and students are encouraged to work at the easiest level. Questions most frequently asked about the program by teachers are answered in this guide. (Author/SA)

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Comprehensive School Mathematics Program

CEMREL, Inc.

Experimental Version

4-82001

CSMP Mathematics for the Intermediate Grades

Part II

Teacher's Guide

General Introduction
Workbooks

U.S. DEPARTMENT OF HEALTH,
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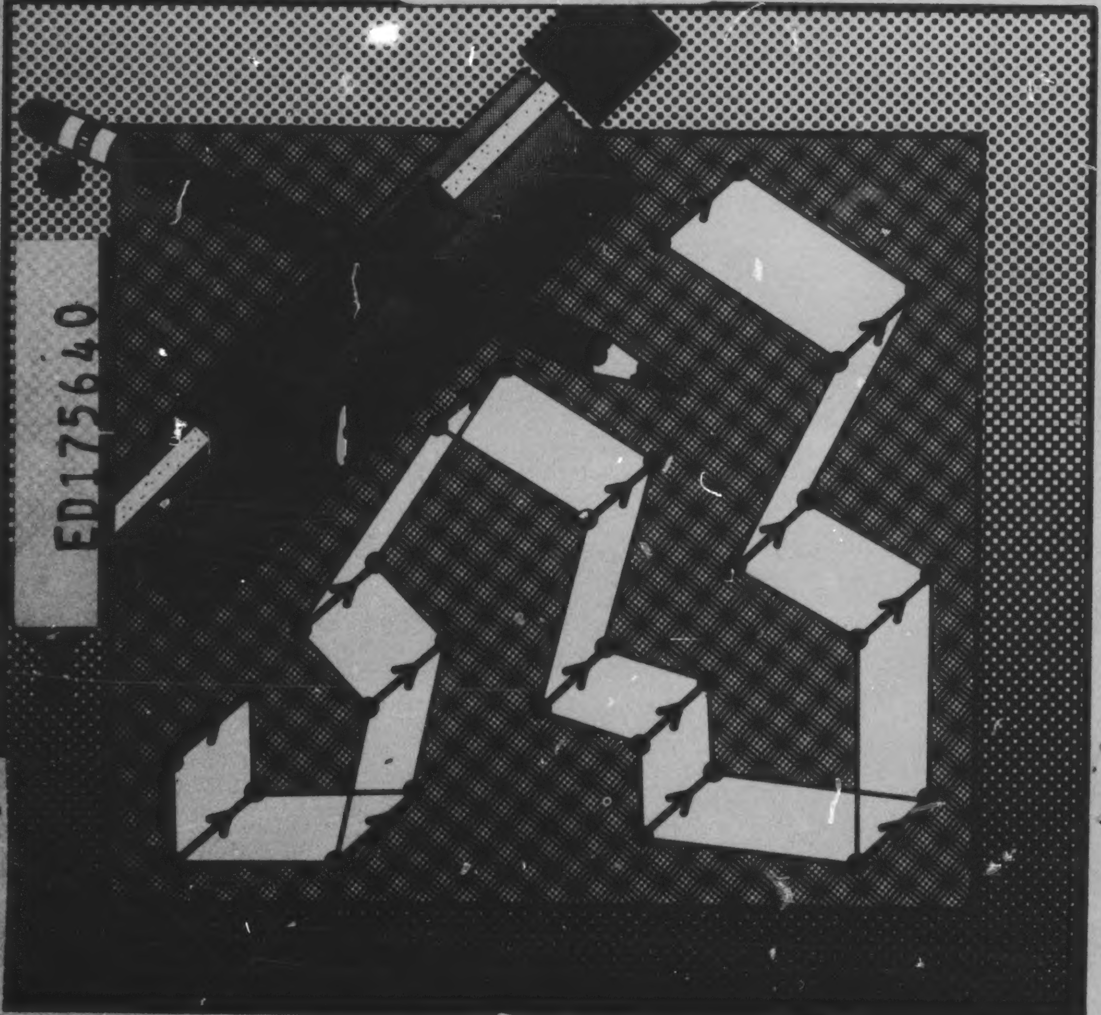

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PREFACE

CSMP is firmly committed to the proposition that the study of mathematics should be an interesting, challenging, intellectual and aesthetic experience for all children. The teachers who will teach its curriculum should be similarly committed to that proposition. The CSMP curriculum expects to truly engage children in the study of mathematics. It expects to call upon their curiosity, their intuition, their powers of reasoning, their imagination, their perseverance, and their organizational skills. We believe that the study of mathematics not only calls upon these faculties of children, but will help to develop them. But the curriculum cannot do the job by itself--there must be teachers who share the basic intellectual values built into the curriculum.

There is an almost infinite variety of uses to which mathematics can be and has been put. Certainly, a reasonable mastery of arithmetic skills is part of the mathematical equipment of a competent, modern adult. Our job as educators is to provide children with a set of mathematical experiences far broader than the acquisition of simple arithmetic skills. It is just as important to expose children to a wide variety of ways in which mathematics can help them to formulate and think about problems. We believe further that the arithmetic skills themselves can be much more successfully taught in the context of a rich and challenging mathematical environment. It is especially important to expose children to a wide variety of mathematical activities in view of the fact that no one can possibly know what career and lifestyle an elementary school child will ultimately want to choose.

It is CSMP's conviction that mathematics is a unified whole and should be learned as such. The organizational basis of the CSMP curriculum is the spiral approach. Thus, the program is sequenced so that students come into contact with each area of content continuously throughout the program. This approach consciously precludes atomizing the content and mastering each bit before continuing to the next. Rather, students encounter repeated exposures to the ideas, building interlocking experiences of ever increasing sophistication.

CSMP believes that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation and independent individual study.

The CSMP materials are constructed to maximize teacher-student and student-student interaction. Children begin immediately to investigate mathematical concepts in concrete situations appealing and meaningful to them at their level of experience.

Individualization at this level is achieved through graded series of workbooks, integrated into the basic sequence of lessons. Due to the spiral nature of the curriculum, each student can engage in a workbook series for a period of from two to four weeks without losing touch with the progress of the class and, at the same time, working his or her way up through the series to the limits of his or her capacity.

Unlike most modern programs, the content is not presented as an artificial structure external to the experience of children and into which they are thrust but rather as an extension of experiences children have encountered in their development, both at the real life and fantasy levels. Children are led through sequences of problem solving experiences presented in game-like as well as story settings. Nonverbal languages give children immediate access to the mathematical ideas and methods necessary not only for solving these problems but also for continually expanding the students' understanding of the mathematical concepts themselves. These languages include:

1. The language of strings. This language mimics the basic method of organizing and collecting data related to problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of sets.

2. The language of arrows. This graphical language models the fundamental process involved in comparing and analyzing sets and operations on them. That is, it deals with the useful and important mathematical notions of

relations and functions including, but not restricted to, permutations of sets, ordering relations and the numerical processes of adding to, subtracting from, multiplying by and sharing equally among.

3. The language of the Papy Minicomputer. The Papy Minicomputer, an extremely effective abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to very sophisticated algorithmic processes, including the standard algorithms for the basic numerical operations.

Through these languages the children are able to apply the basic mathematical ideas effectively and creatively to explore the content directly without wading through the cumbersome, sterile terminology and notation which, in most other programs, pervade the early stages of the curriculum and stand as barriers between the child and the content.

The content is learned in an atmosphere of constant connection with applications, from simple story situations in the early grades to challenging applications to nontrivial simulations of real world problems in the upper grades. The emphasis is at all times on a two level approach to learning: understanding the content and its applications, but equally important, developing the techniques and processes of learning the content. It is the latter form of knowledge which gives the power to apply the former.

The preface above also serves as the Preamble to the Memorandum of Agreement between CEMREL, Inc., and each school district involved in the extended pilot test stage of the CSMP Elementary School curriculum.

NOTE ON AUTHORSHIP

"Teacher's Guide to CSMP Mathematics for the Intermediate Grades, Part II" is the product of the concerted efforts of a number of people over several years' time. This edition was produced during the 1976-77 academic year by the following members of the CSMP staff and includes substantial contributions from the staff associates.

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The lessons in this guide were developed and tried out with two fifth grade classes at the Jackson Park Elementary School, University City, Missouri.

The students, representing the full range of abilities, had been introduced to the CSMP curriculum at the beginning of fourth grade in a one semester program using "CSMP Mathematics for the Intermediate Grades, Entry Module I". The entry program prepared these students to begin the first part of the CSMP Intermediate Grades Program for veteran students during the second semester of the fourth grade.

We are extremely grateful for the daily cooperation and high level of constructive criticism received from Mr. Robert Graham and Ms. Patricia Lee, the regular teachers of the two classes used for developmental purposes. Our greatest motivation for this work was the enthusiasm and steady progress of the students themselves.

GENERAL INTRODUCTION

Comprehensive School Mathematics Program

Mathematics for the Intermediate Grades
Part II

General Introduction

CEMREL, Inc.

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1. The CSMP Viewpoint

The CSMP curriculum is based on the belief that mathematics belongs to that part of the school curriculum which deals with ideas rather than that part which stresses techniques. This is not to say that there are no techniques involved in the learning of mathematics. There are, just as there are in the learning of English. But these techniques, just like spelling and grammar in the learning of English, are not ends in themselves; they are important only in so far as they help children to understand and use some area of mathematics.

What are the objects of mathematical study? For us, they include numbers of various kinds, operations on and relationships between those numbers; geometrical figures and their properties; relations and functions (e.g., the 2 x function, the "less than" relation, etc.) and operations on them.

Among all objects of mathematical study, from the simplest to the most complex, there is one common property: if we separate these objects of study from the real life situations that they model, they are all abstract; that is, they are things of the mind. For this reason, mathematics is, at heart, an intellectual activity. For example, the number 2 represents the phenomenon of twoness, a quality which is present in an endless variety of contexts in life. We can picture 2 by two dots, or by a stick two centimeters long, and these representations are certainly useful in suggesting to our minds what may or may not be true about the number 2. But ultimately the number 2 is accessible to us only through our minds. In the same sense, a geometrical line is neither a pencil line nor a stretched string nor a ray of light, but is an abstraction which may be pictured in many ways.

This is surely an adult point of view. But it is one that should be understood and taken into consideration by those who develop mathematical materials for children and those who teach the materials to children.

Since mathematical objects are ideas, one of the important ways we explore the properties of and relationships among such objects is to think about them. It is for this reason that the growth of the ability to reason plays such an important role is mathematics. We strongly argue, however, that in mathematics the development of the art of reasoning goes hand in hand with the growth of imagination, ingenuity, and intuition. The really important thing at the elementary level is that mathematical objects, their properties and their interrelationships be thought about and discussed in various suggestive and interesting situations.

Seen in this perspective, elementary arithmetic then takes the form of "adventures in the world of numbers". Individual numbers such as 0, 1, 2, 100, $\frac{1}{5}$, $^-7$, $^-\frac{4}{9}$, 3.72, $^-1.006$, and so on can assume definite personalities of their own in this world. The teaching of arithmetic shifts its emphasis from an obstacle race for mastery of the standard algorithms to the stimulation of exploring the world of numbers. Skills such as balancing a checkbook have their place in real life but are of absolutely no interest to a fourth or fifth grader. It is unrealistic to view such activities as goals of an elementary school mathematics education. Far more genuinely rewarding are the stretching of the powers of imagination and the challenging of the mind. Facility then grows naturally, along with knowledge and understanding. Necessary skills become by-products of such an educational experience.

Such a curriculum requires a special kind of pedagogy. The pedagogy employed should include teaching strategies which encourage creativity and allow freedom to explore but still foster sound intellectual habits. Attitudes toward student achievement that allow for intellectual and emotional growth and respect the individuality of each student are essential to the successful attainment of the goals of this type of curriculum.

The pedagogy which CSMP has developed in this spirit can be described as a "pedagogy of situations".[*] The introductions to the various strands of the CSMP Intermediate Grades Program and those to the chapters within them contain many examples which illustrate aspects of this pedagogy. All these examples are taken from the lessons themselves, and a prime source of "pedagogical situations" is the lessons of the program. At the risk of oversimplification, a pedagogy of situations may be described as one which is based on the belief that learning occurs in reaction to the experience of confronting a situation (real, simulated, or imagined) that is rich in consequences, worthy of confrontation, and has genuine intellectual content. For example, we can tell a story about some children in a playground.

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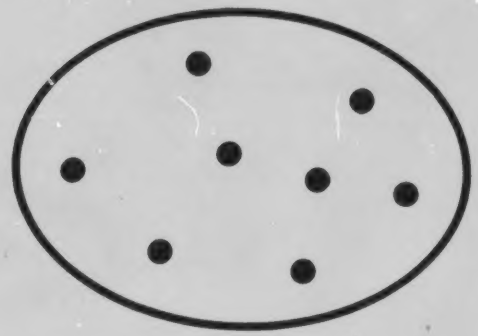
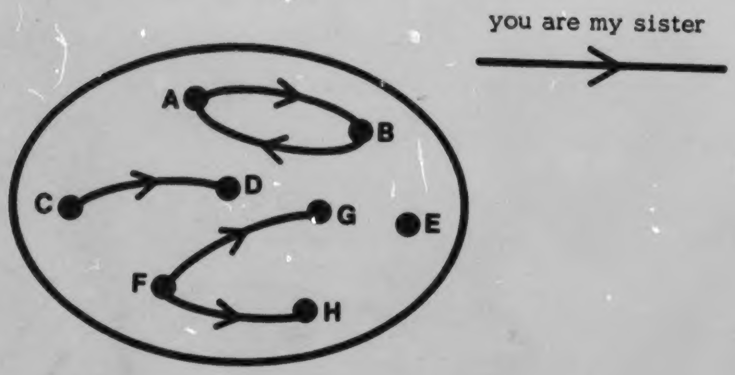
[*] This phrase is due to the mathematician and psychologist, Caleb Gattegno.

As we tell the story , we draw this picture.



Each child in the playground is represented by exactly one dot, and each dot represents exactly one child. (There is no need to draw pictures of the children; indeed, such pictures would not serve our purposes at all. Even young children are quite able and willing to accept a dot as a symbolic representation for a child. On other days and in different situations they will accept the dot as representing a duck in a pond, or a marble in Johnny's collection, or a number, or whatever else you or they decide.) The children in the playground are playing a game: each child points to his or her sister(s) and says, "You are my sister." As we continue the story we draw arrows as indicated. (We label each dot here with a letter for easy reference as we discuss the situation.) So, for example, C is pointing to D and saying, "You are my sister."



What kind of comments will young children make when involved in this situation? Here are a few typical responses.

1. There are eight children in the playground.

2. D is C's sister.

3. D is a girl (Why? Because D is someone's sister.)

4. E has no sisters--at least no sister in the playground.

5. A and B are sisters and therefore both are girls.

6. There are two arrows missing. (Which? G and H are sisters of F and so they should be pointing to each other.)

These two arrows are added to the diagram and the children are now told that all the children that can say "You are my sister" have done so. That is, all the red arrows that belong have been drawn.

7. At this point some child in every group will almost always suggest "C is a boy!" If we ask "How do you know that?", we get an answer that can be summarized in adult language as: Suppose C were a girl. Then C would be D's sister. But there is no arrow from D to C. So the diagram tells us that C is not D's sister, and hence C is not a girl. So C is a boy.

8. There are children from four different families in the playground.

9. C is D's brother.

The students are now asked to draw arrows for "You are my brother" in a different color. They draw "You are my brother" arrows from D to C, G to F, and H to F.

In this example, the children react to a situation that intrigues them. They grapple with problems of symbolization and the relation of a symbol to the thing it represents. Arguments are given that are close to those which are the stuff of mathematics. Although adults may protest that it is not a "real" situation, it _is_ real to children, who can often involve themselves in and feel at home with "pretend" situations more readily than with many "real" situations. On the basis of several years' experience in a wide variety of classes, we can say with confidence that such situations

- involve children at a personal level (they are interested in the situation presented, and sometimes want to dramatize it or devise similar or related situations);
- present an intellectual challenge that is accessible to a broad range of abilities (even the slowest students understand the challenge and can learn something from their experience in confronting it; some may even be able to realize the full learning potential of that challenge);
- suggest opportunities for creativity (some children are motivated by such situations to generalize from their experiences or to make conjectures to be tested in similar situations);
- provide experiences that have mathematical content. In the example of the "brother-sister situation" above there are two important mathematical activities at work. One is the classification of objects according to certain attributes (brother-sister, boy-girl, same family). More importantly, the experience concerns, at a very elementary level but in a natural setting, the process of deducing certain conclusions from given data and at the same time realizing the limits of deduction in the situation at hand.

The CSMP curriculum fulfills what we consider to be the fundamental role of mathematics in the elementary school by presenting a large number of varied yet interrelated situations that provide the experiences out of which mathematics grows. Two questions immediately arise. How do students learn necessary "number facts" via a pedagogy of situations? And how can we "grade the achievement" of a child as he or she reacts to a situation?

Believe it or not, the "number facts" are among the things which students learn almost inadvertently under a pedagogy of situations. Numbers take an almost living-form and have personalities of their own. Their interrelations are the subject of many stories in which numbers talk to each other and react to each other. In confronting some of these situations it is necessary to make various calculations, and calculation which serves some higher, more motivating purpose presents nowhere near as many difficulties as does calculation for its own sake. There are further remarks concerning calculation scattered through the latter part of this introduction, and also in the introductions that appear in the World of Numbers strand.

As for "grading of achievement", CSMP takes the position that learning often stops when the purpose of a lesson or sequence of lessons is the mere mastery of skills. Every child should learn something from each situation; otherwise it is not a fruitful situation and should be revised or eliminated. But to measure what or how much has been learned in any one situation is probably not possible. Learning is not a linear process. It occurs in quantum jumps that take place at times when you least expect them. It is possible to monitor the progress of students, and you all have your own ways of doing this. But who is to say at what precise time everyone in a class is supposed to have gained a particular skill or concept from his or her experiences? We comment further on this matter in the next section of this General Introduction.

A more substantive question is: How can we present situations rich in mathematical content before children have the verbal equipment and the computational skills to deal with them? It is presupposed by many people that children must be able to read and to calculate before they can tackle any mathematical situation other than trivial ones. Yet if one analyzes the types of thinking and the few basic concepts underlying elementary school mathematics, one finds that there is nothing esoteric or even difficult about these thinking modes or these concepts. What has been difficult for students in the past is the language in which these modes of thinking and these concepts are traditionally couched. For some reason, in our natural languages these modes of thinking are difficult to communicate precisely. That is why mathematicians devise languages of their own, languages that minimize the ambiguities of natural languages. Part of the problem in the past has been that elementary school children have found it difficult to deal with such tricky uses of language at a time when they do not yet have a firm command of their natural language.

On the other hand, since the underlying ideas and modes of thinking are inherently simple, there is no reason why very young children should not have the pleasure of such thinking and of exploring mathematical ideas. All that is needed are languages with which to express such ideas and to communicate the thinking that is generated by the ideas. These languages must be very simple, essentially nonverbal, and precise.

CSMP uses three such languages that fit the criteria and that have been highly successful in allowing students to do the kind of thinking that we described above. We call them the language of strings, the language of arrows, and the language of the Papy Minicomputer.

The first two, as their names imply, are picture languages: brightly colored strings and dots enable precise descriptions of and the accurate recording of thinking about classifications of objects; colored arrows between pairs of dots stimulate thinking about relations between objects. For detailed descriptions and discussions of these two languages you are referred to the introduction to the Languages of Strings and Arrows strand.

The third language is based on a simple abacus called the "Papy Minicomputer". Using this language, one may represent all decimal numbers (positive or negative) in such a way as to encourage creative thinking about the nature and properties of those numbers. With the support provided by the Minicomputer, the children, even at an early age, have the opportunity to develop excellent mental arithmetic skills and to gain sophisticated insights into the world of numbers. The ability to perform mechanical computation is one of the incidental by-products of such an approach to work with numbers.

2. Program Description

a) Description of Materials

The CSMP Intermediate Grades curriculum is designed as a program for students in grades four, five, and six. There are six parts to the curriculum, each corresponding to one semester's work. This is Part II of the six. It follows Part I of the CSMP Intermediate Grades Program.

In addition there are three Entry Modules, one for each grade level. As their names imply, each Entry Module is designed to facilitate transition into the CSMP Elementary School curriculum.

The materials of Part II of the CSMP Intermediate Grades Program require some explanation. This collection of six books which are directed specifically to you, the teacher, is known as the "Teacher's Guide to CSMP Mathematics for the Intermediate Grades, Part II". (By way of abbreviation, in what follows we shall refer to this collection simply as the "Teacher's Guide".) With the exception of this book, "General Introduction", each book of the Teacher's Guide addresses itself to a single area of the curriculum, and each of these areas is known as a "strand". The four content strands are:

- the World of Numbers. The abbreviation for this strand is "N".
- the Languages of Strings and Arrows. The abbreviation for this strand is "L".
- Geometry and Measurement. The abbreviation for this strand is "G".
- Probability and Statistics. The abbreviation for this strand is "P".

The six books comprising the "Teacher's Guide" are bound into three volumes, with a separation page inserted between the various books of a given volume. This volume contains both this General Introduction as well as the Workbook strand (which is abbreviated "W"). One of the other two volumes contains the World of Numbers strand, and the other contains the Languages of Strings and Arrows strand, the Geometry and Measurement strand, and the Probability and Statistics strand. Each of these six books has its own table of contents and is paginated individually.

The content and Workbook strands have the following arrangement of material.

- A table of contents. This enables you to locate whatever you need in that particular strand.
- A suggested schedule for the teaching of the lessons in that strand. This provides you with one among the many possible ways of organizing the material in the strand. It also includes for each suggested lesson a list of the materials you will need when you teach that lesson. See Section 3 of this General Introduction for a discussion of the way in which you should view this suggested schedule.
- A suggested chronological order of the lessons in that strand. This provides any easy, quick reference uncluttered by extraneous information. It shows you in which order the lessons described in the strand will be taught if you follow the suggested schedule.
- An introduction to the strand. This provides you with information explaining why certain material finds its place in that strand, what are the overall goals of study in that strand, and how this material fits in with that which is described in other strands.
- Suggestions for the planning and conducting of lessons in that strand. These include lesson descriptions, suggestions on the use of worksheets and manipulative materials, general remarks relating to certain sequences of lessons, etc.

The suggested lessons are grouped in chapters, all the lessons described in the same chapter having some particular feature(s) in common. Because of this commonality of purpose, it is possible to precede each chapter by an introduction which indicates where the lessons in that chapter are going and on occasion gives a few more details concerning the mathematical underpinnings of the lessons described in the chapter than are contained in the lesson descriptions themselves.

Many of the suggested lessons require student worksheets for individual work. Classroom sets of these worksheets are printed and packaged separately from this guide. However, an answer key for each of the worksheets is included in the guide. Worksheets are numbered consecutively within each strand and their answer keys can be found at the end of the lessons for which the worksheets are needed. For example, an answer key for Worksheet P03 is at the end of Section 1.4 of the Probability and Statistics strand since it is suggested that this worksheet be used with the lesson described in that section.

In order to accommodate the various ability levels in a given class, the worksheets generally are written at several levels of difficulty denoted by stars. "*" represents an easy worksheet, "**" represents an average level worksheet, and "***" and "****" represent difficult level worksheets. All students should begin with the "*" level worksheets for a given lesson or sequence of lessons and proceed as far as possible through the other levels. In classes containing the usual range of abilities, most of the students should succeed with the * worksheets; the ** worksheets should be appropriate for about two-thirds of the class; and approximately one-third of the students will find the *** and **** worksheets challenging and enjoyable to do. No one should ever be required to attempt all the worksheets provided for a given lesson. Each student should progress as far as his or her abilities allow.

There are various other items which are (or could be) used in the program. Among them are the following: various manipulative materials such as translators, marbles, grid boards, and so on (these are described in more detail in the lessons in which they are used); several workbooks, storybooks, and story-workbooks (these are described in more detail in Sections 2 (b) and (e) of this General Introduction, and in the introduction to the Workbook strand); a book entitled "Math Play Therapy", which includes a collection of suggested small group activities that could be instrumental in stimulating an open and positive attitude toward mathematics on the part of those of your students who find the subject difficult. See the introduction to "Math Play Therapy" for suggestions as to its use in the program.

b) Role and Use of Workbooks

It is suggested that approximately one day each week be scheduled for students to do individual work in designated workbooks. The workbooks are designed to review ideas encountered in the content strands and to apply these ideas to new situations posed as stories. Many of the workbooks are written at more than one level of difficulty in order to accommodate differences in students' abilities and interests.

More detailed descriptions of the workbooks, their purposes, and answer keys are given in the Workbook strand. There you will also find suggestions for keeping records of student progress.

c) The Spiral Development

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson is an end in itself. It is neither intended nor expected that every student will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations and strands from day to day rather than to continue with one type of situation or piece of content until so-called mastery has occurred. Thus, we suggest that the lessons be scheduled so that a different topic is studied each day of the week. In a typical week, for example, your class might work with numerical games one day, probability the next day, workbooks the third day, numerical relations the fourth day, and geometry the fifth day.

This spiral development, in which a student experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging, reflects the fact that learning is not a linear process. Not only do different children learn at different times and different rates, but even on an individual level learning can occur in quantum leaps that take place at times when you least expect them. The spiral development gives each student a new chance to catch on to an idea at each turn of the spiral.

Of course, acceptance of the CSMP pedagogy requires a certain act of faith on your part. If you have been accustomed to staying with a given topic until everybody (or nearly everybody) in the class knows it "cold", then you may be initially very reluctant to leave a lesson at the end of the time suggested, believing that several students "haven't got it yet". By keeping the spiral idea in mind, you may become more comfortable with these time schedules. Although it may appear to you that little learning is taking place in a sequence of lessons, you may be surprised that suddenly even your slowest students are understanding the ideas when they encounter them later in the spiral. Whereas with a mastery approach, presented in a purely individualized mode, slow students often spend most of the school year trying to "master" low level skills (with generally poor results), the spiral approach gives these students the opportunity to benefit from the richness of the situations presented throughout the curriculum. In this manner students are allowed to become involved in interesting and enjoyable situations which are modeled by important mathematical notions. They are not deprived of a variety of experiences just because they haven't mastered every bit and piece along the way. Therefore, in the long run, their interest in, understanding of, and even facility with mathematics has a much better chance to develop.

d) Evaluation

On the basis of the preceding discussion, it should come as no surprise, as stated earlier, that CSMP takes the position that learning often stops when the purpose of a lesson or sequence of lessons is the mere mastery of skills. Furthermore, we do not believe it is desirable or reasonable to attempt to assess student mastery of skills from day to day or even from week to week. As we said before, learning has a habit of occurring when you least expect it, so how does one determine when everyone in a class should have mastered a specific skill or attained understanding of a particular concept as a result of his or her experiences? We know of no method for measuring what or how much has been learned from any one situation except in the most trivial cases. On the other hand, teachers, parents, and even the students themselves can benefit from some continuous monitoring process that records involvement in activities and assesses progress through the program. Indeed, it is possible to monitor your students' progress without contradicting the statements made above; all of you have your own ways of doing this. Such a process is suggested in the introduction to the Workbook strand. Of course, you should also continue to use any monitoring devices with which you feel comfortable.

As far as traditional computational skills are concerned, the standardized test data collected during the extended pilot test stage of the development of this curriculum indicate that CSMP classes generally do better than average on the items which test "basic" skills.

About 75% of the lessons for the CSMP Intermediate Grades Program involve numerical concepts. The situations do not involve just the integers (as is generally the case at this level), but introduce the students to many other citizens of the world of numbers at as early a stage as possible. You should not infer from this that your students are expected to have mastered the arithmetic of the whole numbers, the integers, the decimal numbers, and the rational numbers before they are ready for Part III of the CSMP Intermediate Grades Program. On the contrary, it means that the students will become more and more friendly and at ease with the various types of numbers and will develop their own methods for calculating with them long before some standard mechanical algorithms are introduced.

It is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying, and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of natural synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

We therefore strongly urge you to follow the suggested schedule of lessons, teaching the algorithms as they come, but not giving them more emphasis than they deserve. Skill in mental arithmetic and estimation, however, is extremely important in our opinion and should be promoted at every opportunity.

While CSMP is opposed to the philosophy underlying standardized testing, we realize that students as well as teachers are under extreme pressure to perform in certain ways with regard to such tests. If your students or their parents express concern about the apparent lack of emphasis on mastery of standard algorithms (such as long division by three or four digit numbers), they should be reassured that CSMP students have a definite advantage when it comes to applying mathematics to novel situations, strategic thinking, organizational ability, and understanding of the methods of mathematics. Because CSMP feels it is wrong for such tests to dictate emphasis on the mastery of standard algorithms and that drill for algorithmic mastery for its own sake defeats CSMP's purposes, you will find that even after algorithms have been introduced into the CSMP curriculum the role they play will be as tools for doing other interesting things. Therefore, if you feel pressured by the tests or otherwise to put more emphasis on drill activities, then we suggest you use any supplementary materials that have proved successful in the past to provide this extra practice. We leave scheduling choices for these kinds of activities to your judgement.

e) Role and Use of Storybooks and Story-Workbooks

Among the instructional materials for the CSMP Elementary School curriculum is a collection of mathematical storybooks, STORIES BY FREDERIQUE. This collection consists of 24 stories for young people, each of which is presented in the format of a newspaper for children. Nearly every story is printed in color on tabloid size ($17\frac{1}{2} \times 11\frac{1}{2}$)* newsprint and is intended to appeal to youngsters in the 5-14 age range. The stories are completely independent of each other and need no external explanation. They may be used for individual or collective reading, at home or at school.

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*Six of the stories are printed on $8\frac{1}{2} \times 11$ newsprint rather than the tabloid size.

Each of the stories is portrayed by means of the mathematical languages of arrows, dots and strings which reinforce the ideas of the story. Three of the stories (TWO BY TWO, THE WEIRD STORY OF 24, and SUMMER SCHOOL IN THE OLD DAYS) also use the language of the Minicomputer. The amazing versatility of these nonverbal languages is exploited to the full.

Young readers are personally involved in each story. They have to answer questions and use their imaginations to invent additional adventures for the characters in the story. They learn that there is room in the make-believe world of mathematics for exercising the aesthetic sense and for expressing emotions.

Twelve of the stories are intended for very young children (5 to 8 years of age). Here the focus is on the anecdotal aspect of the story. One of the goals of these twelve books is to make the children familiar with nonverbal mathematical languages and to leave them with the impression that these languages are as natural and useful as the usual language.

Nine of the remaining stories concern numbers and their relationship with the world. These particular stories are about a boy and some of his friends who just happen to be numbers. The personification of numbers introduces the "games" of mathematics into one of the most important domains of childhood -- the world of fantasy. Free from the usual constraints so often placed upon them, the numbers invent marvelous games and dance to the tunes of vividly colored arrows. These games are never frivolous. There are rules to be followed and as a result, orderly patterns emerge as the numbers play or dance.

Of the remaining three books, two are detective stories introducing some interesting mathematical problems, and one emphasizes the various and very different aspects of a simple straight line.

The titles of the books in the collection and the intended age range for each story are:

Ages 5 to 8

THE PLAYFUL NUMBERS
81 ROSES
I AM A VERY HAPPY BOY
ONE OUT OF SEVEN
THE HAPPY PUPPET
TWO BY TWO
THE LITTLE DREAMER
THE LITTLE DONKEY
THE MAGIC BOX
THE BABY IS BORN
THE WEIRD STORY OF 24
THE OLD SHOEMAKER

Ages 8 to 12

WHERE'S MY NOSE?
SINGING FRIENDS
DANCING FRIENDS
THE LIVING LINES
I AM NOT MY NAME
NABU WINS AN AWARD
THE SQUARE TRAP
SUMMER SCHOOL IN THE OLD DAYS

Ages 10 to 14

THE HIDDEN TREASURE
A VERY STRANGE NEIGHBORHOOD
ELECTION IN THE NUMBER WORLD
A VALENTINE MYSTERY

In Part II of the Intermediate Grades Program, there is one lesson explicitly involving one of these books: NABU WINS AN AWARD. In addition two copies each of the following storybooks are provided: TWO BY TWO, WHERE'S MY NOSE?, SINGING FRIENDS, DANCING FRIENDS, THE LITTLE DONKEY, THE LIVING LINES, and I AM NOT MY NAME. They should be put in the class library for students to read whenever they like. These are books that were in the third grade program and that students may enjoy reading from time to time. If Part II is being used in the second semester, following Part I in the first semester, these story books are not included with the Part II materials since they were provided with Part I materials.

Besides the collection, STORIES BY FREDERIQUE, another collection, THE CSMP LIBRARY, has been developed as part of the instructional materials for the CSMP Elementary School Program. This is a series of story-workbooks which combines the spirit of the storybooks and the workbooks into a single integrated experience for the children, engaging them in reading about and doing mathematics in stimulating, challenging story situations. Two of these, CLINTON STREET (Books A and B) and A STRANGE COUNTRY are incorporated into the Workbook strand.

f) Role and Use of Hand-Calculators

The electronic hand-calculator, if used creatively, can have a remarkably beneficial effect on the students' interest in and understanding of arithmetic, and its use can be a contributing factor in the improvement of their capabilities in that area. The reasons for this are many and varied, but among them are certainly the following:

- By removing the burden of mechanical computation from the children's shoulders, the calculator frees them to concentrate on the thinking that is a necessary part of the solution to any meaningful problem.
- The instant feedback provided by the calculator enables the children to indulge in making and testing conjectures, an area into which they would not otherwise venture because the quantity of computation involved would be too forbidding.

This pedagogical tool is particularly useful in the introduction of new numerical notions such as negative integers, decimal numbers, multiples of a given whole number and so on, because through suitably chosen, rich situations, these concepts can be made to arise in a natural way.

Two rechargeable, battery powered hand-calculators are recommended materials for Part II and are available from CSMP.* A recharger unit that permits the hand-calculator to operate on alternating current while simulteneously recharging the calculator's batteries is also available. To prolong the useful life of the nickel-cadmium batteries the following precautions are recommended.

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* Some Part II lessons require two hand-calculators in the class. If you already have calculators, be sure to check your calculators for algebraic logic and constant mode (as explained on the following page).

1. Charge the batteries for a full five hours with the calculator off before making extensive use of the calculator. This applies to the initial charge or if the calculator has not been used for an extended period of time -- as long as six months.
2. Recharge the batteries as soon as the display is noticeably dimmer. Continued use of the calculator beyond this point can shorten the useful life of the batteries.
3. The batteries should not be continuously charged for a period longer than 72 hours.

While the two hand-calculators are adequate for the lessons in Part II, the ideal situation would be for each student to have one. Encourage the students to bring calculators from home. The hand-calculators that are available from CSMP have algebraic logic and function keys which are permanently in a constant mode. You should check to be sure that any calculators brought in by students also have these characteristics.

- Algebraic logic. This means that the calculator responds to instructions given in the order in which they are usually written. To check that this is the case, press $\boxed{2}$ $\boxed{+}$ $\boxed{3}$ $\boxed{=}$. If you have "5" on the display, then all is well.
- Constant mode. Some calculators have a switch so that the calculator can be set either in a constant mode or in a chain mode. If your calculator lacks a switch, it is quite likely that the calculator has an automatic constant. For calculators with an automatic constant, the first number entered during multiplication is usually the constant, while the second number entered during the other three operations (addition, subtraction, division) is usually the constant.

  To check your calculator for the constant mode operation of $\boxed{+}$ is a simple procedure. Press $\boxed{0}$ $\boxed{+}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. If the number on the display is increasing by two at each depression of $\boxed{=}$, then $\boxed{+}$ is operating in a constant mode. Check the constant mode operation of $\boxed{-}$ in a similar fashion.

To check the constant mode operation of $\boxed{x}$ (where the first number entered will be the constant) press $\boxed{2}$ $\boxed{x}$ $\boxed{1}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ and so on. If the number on the display doubles at each depression of $\boxed{=}$, then $\boxed{x}$ is operating in a constant mode.

While it would be possible to use calculators without these characteristics in the lessons, the adjustments that would be required in the instructions make the calculators unsuitable.

Finally, one last thought concerning the idiosyncrasies of some hand-calculators: in several lessons the hand-calculator relations $\boxed{+}$ $\boxed{n}$ $\boxed{=}$ ... and $\boxed{+}$ $\boxed{=}$ ... are explored quite extensively. Even if your calculators have algebraic logic and the constant mode, it may be necessary for you to check to see if your calculators operate in the way that is described in the lessons.

If you find that your calculators do not operate in this manner, then try letting

$$\boxed{+}\ \boxed{n}\ \boxed{=}\ \ldots \qquad \text{mean}$$

$$\boxed{+}\ \boxed{n}\ \boxed{=}\ \boxed{+}\ \boxed{n}\ \boxed{=}\ \boxed{+}\ \boxed{n}\ \boxed{=}\ \ldots$$

and

$$\boxed{+}\ \boxed{=}\ \ldots \qquad \text{mean}$$

$$\boxed{n}\ \boxed{+}\ \boxed{0}\ \boxed{=}\ \boxed{+}\ \boxed{n}\ \boxed{=}\ \boxed{+}\ \boxed{n}\ \boxed{=}\ \ldots$$

where n is some starting number.

3. Guidelines for Organizing the Program

The content of the lessons in Part II is designed to afford you the utmost flexibility in teaching the program successfully. It is a truism to say that your students this year are very different, both individually and as a class, from the students you taught last year, and indeed from other students in any other CSMP class. And your own teaching style and interests are probably very different from those of your colleagues. It is difficult for those of us who develop curricula to guess correctly as to which topics will be very easy for your students and which lessons will require more instructional time. You are the only one who will be able to judge that as you begin using the materials. This program is not meant to be a static sequence of exercises to be followed ridlly; the schedule and design of lessons in this guide should be regarded as suggestions; they simply reflect our own experience teaching the materials. You and your students will make the program dynamic as together you explore the mathematical ideas and follow your own interests and inclinations.

a) Use of Schedules

If this is the first time you are teaching a CSMP class and you would like specific suggestions for organizing the lessons, we offer some guidelines and various suggested schedules of lessons. Please remember that our suggested sequence is but one of many that could have been assembled. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate.

The master schedule which is included at the end of this section was developed for a hypothetical fourth grade class of average ability, and it illustrates some important ideas about organizing the program. You will notice that several different strands are scheduled each week and that even within a strand the topics are interspersed. This was done to carry out the idea of a spiral curriculum. If you organize your own schedule it is best to follow a similar pattern rather than to spend two or three full weeks, for example, "covering" one particular topic such as composition of relations or decimal numbers. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Workbook strand, and the other two days should be shared by the P, G and L strands.

Within each strand (or part of a strand) we provide a more detailed schedule aimed specifically at that strand (or part of a strand). In the case of the P strand which has only five lessons, the schedule shows how these lessons are interspersed among L and G lessons. The schedule which appears in each strand book has the lessons of another strand shaded over so that only the lessons of the strand itself stand out. In each schedule, a block represents one full period of approximately 40-45 minutes duration. It may very well be, however, that you and your class will spend more than one period on some lessons. Exercise your own professional judgment in deciding when to stop one particlar lesson and when to take it up again. Just be sure that you do not insist that every single child understands every single aspect of a lesson before you go on. It is better to stop a lesson at a point when student interest is high and a problem remains unsolved than to teach so methodically that everyone in the class understands everything, but is completely bored.

Inside each block of the detailed strand schedules, the lesson of the day is designated in several ways. First, there is listed a number which indicates its suggested chronological order among the lessons in that strand. Hence, "5N" indicates that the lesson in question is suggested as the fifth lesson in the N (World of Numbers) strand. This identifying number also appears in the upper outside corner of each page in the description of that lesson, thus enabling a lesson to be located simply by flipping through the pages of the strand book. Second, you are told on which page of the strand book the description of the lesson in question begins. Third, as a double-check, you are given the section number and title of the lesson. Next to each block in each schedule there is a list of the materials you and the students will need in order for you to be able to teach the corresponding lesson.

Please bear in mind that the model schedule was designed for a hypothetical class with the usual range of abilities. In order to be successful with the program, you should adjust the schedule to the ability, maturity, and interests of your class. For example, you may wish to extend a lesson or a workbook series for several additional periods beyond those scheduled.

b) Use of Adjustment Days

From time to time an adjustment day appears in the schedule. Since we have intended that Part II would be the mathematics curriculum for one semester of the Intermediate Grades and a usual school semester is 18 weeks, the schedule has been designed to indicate 18 weeks of lessons. However, it is also usual for there to be days when no mathematics lesson is taught; e.g., parent-teacher conference days, field trip days, snow days, and so on. These adjustment days should account for such non-class days, and should also give you opportunities to do many kinds of things. You could use an adjustment day to review a lesson you feel needs review, or to complete an unfinished lesson, or to extend some interesting ideas from completed lessons. You could also use an adjustment day to give your students time to become more familiar with the manipulative materials in the program, such as the translators or the coordinate grids or storybooks. The students may especially enjoy playing games such as Minicomputer Golf or Minicomputer Tug-of-War or Guess My Rule or the String Game, and an adjustment day could give them another opportunity to play a game. You may wish to use an adjustment day to do some mental arithmetic exercises, or to teach a topic in mathematics you think would be especially interesting or useful to your class.

SUGGESTED SCHEDULE OF LESSONS

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART II

WEEK	The World of Numbers	Language of Strings and Arrows Probability and Statistics
1	N1.1 Review: Minicomputer Golf	L1.1 Composition of Relations
2	N2.1 Review of Multiplication	L2.1 The String Game
3	N3.1 Decimal Numbers and the Minicomputer	L2.2 String Game Analysis #1
4	N3.2 A Detective Story about Zip	ADJUSTMENT DAY
5	N3.3 Decimals and the Number Line #1	L1.2 Functions
6	N4.2 Telephone Game #2	L2.3 Logical Thinking #1
7	N6.1 Subtraction #1	L1.3 With and Against
8	N3.4 Decimals and the Number Line #2	L2.4 String Game Analysis #2
9	N5.2 Guess My Rule #2	L2.5 String Game Analysis #3

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART II

Workbooks	The World of Numbers	Geometry and Measurement Probability and Statistics
A Set of Problems No. 1	N1.2 Review: Detective Story	ADJUSTMENT DAY
A Set of Problems No. 1	N2.2 10 x and ÷ 10	G1.1 Length #1
A Set of Problems No. 1	N2.3 Composites of □ x and 10 x	G1.2 Area #1
Arrow Roads	N4.1 Telephone Game #1	G1.3 Area #2
A Set of Problems No. 2	N5.1 Guess My Rule #1	P1.1 Making a Fair Choice
A Set of Problems No. 2	N2.4 Multiplication Algorithm #1	P1.2 Probabilities
A Set of Problems No. 2	N7.1 The Relation, "Is A Positive Divisor Of"	G2.1 Parallelism #1
Clinton Street	ADJUSTMENT DAY	G2.2 Parallelism #2
Clinton Street	N2.5 New Checkers on the Minicomputer	G2.3 Parallelism #3

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART II

WEEK	The World of Numbers	Language of Strings and Arrows Probability and Statistics
10	N3.5 Newtonian Quiz #1	ADJUSTMENT DAY
11	N9.1 Fractions #1	P1.3 Fair Game? #1
12	N10.1 Binary Abacus	P1.4 Fair Game? #2
13	N6.2 Subtraction #2	L1.4 Arrow Messages #1
14	N3.6 Multiples of Decimal Numbers	L2.6 Logical Thinking #2
15	N7.3 Hand-Calculator Relations #2	L2.7 String Game Analysis #4
16	N3.7 Newtonian Quiz #2	L1.5 Arrow Messages #2
17	N2.8 How Many Ways?	L2.8 String Game Analysis #5
18	N9.3 Fractions #3	ADJUSTMENT DAY

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART II

Workbooks	The World of Numbers	Geometry and Measurement Probability and Statistics
Clinton Street	N8.1 Division #1	G1.4 Length #2
Clinton Street	N7.2 Hand-Calculator Relations #1	ADJUSTMENT DAY
A Set of Problems No. 3	N2.6 Multiplication Algorithm #2	G2.4 Parallelism #4
A Set of Problems No. 3	ADJUSTMENT DAY	G1.5 Area #3
A Set of Problems No. 3	N8.2 Division #2	G2.5 Parallelism #5
Nabu Wins An Award	N2.7 Multiplication Algorithm #3	P1.5 Fair Game? #3
A Strange Country	N9.2 Fractions #2	G2.6 Parallelism #6
A Strange Country	ADJUSTMENT DAY	G2.7 Parallelism #7
A Strange Country	N7.4 Hand-Calculator Relations #3	G1.6 Area #4

4. Commonly Asked Questions about the CSMP Program - and Suggested Answers.

As you teach the CSMP curriculum (or even as you prepare to teach it) there will surely be some basic questions that come to your mind. They are the kind of questions that not only you as a teacher, but also parents and supervisors, will ask about any curriculum that appears to be different from the usual program.

The following questions and answers are listed as an aid to you in understanding our curriculum and in responding to those who request information about it.

1. Q: For what sort of student is the CSMP elementary curriculum designed?

A: It is designed for all children in the schools. Each learning situation is designed to involve all students at their own levels in such a way that the slowest can succeed in dealing with some aspects of the situation that challenge them. In fact, it has been found that such a "pedagogy of situations" (see Question 2) is particularly effective with the so-called slow learners, and recent work suggests it might be effective also with educable mentally handicapped children.

2. Q: Briefly, how does CSMP differ from traditional mathematics program?

A: The core content of CSMP is not revolutionary. The program aims to inculcate notions and facts which have always been considered important in mathematics. The major differences are in method. Traditional teaching takes pride in disciplining the student to be submitted to the usual algorithms. With CSMP the student is involved in real mathematical activity, with understanding at his or her level, so as to master the situation.

For that purpose CSMP uses new teaching aids which are essential nonverbal languages and which are themselves both pure and applied mathematics. These mathematical objects (graphs, arrows, etc.) are used as aids rather than as ends in themselves.

A pedagogy of situations is employed to permit the pupil to participate with creative freedom in the learning process. The student is faced with situations under which, by spontaneous reaction, he or she is led to an important idea in mathematics. Thereby, the child learns to mathematize situations, which is of fundamental importance whenever mathematics is applied.

CSMP also makes use of a spiral approach rather than the mastery approach found in most mathematics curricula. (See Question 13)

Q: Isn't this the "new math"?

A: There is a current popular belief that all school mathematics today is either "new math" or "old math". Neither of these labels is well defined. "New math" usually denotes the programs that heavily emphasize the teaching of sets; introduce technical mathematical language for concepts such as "commutativity", "associativity", "distributivity", and so on; and that teach "concepts" before "skills". The "old math" refers to programs of the "good old days": memorization of the multiplication tables and complete emphasis on paper and pencil drill in calculations.

The CSMP curriculum is definitely not the "new math" nor the "old" by any definition. It uses no technical language, yet achieves the understandings sought by the "new math". It uses no repetitive drills, yet elicits the ability to calculate stressed by the "old". But these achievements are just by-products of a more pervasive goal: to help children use their curiosity, their intuition, their powers of reasoning to formulate and think about interesting problems and situations.

Q: Is the student really studying mathematics in the CSMP curriculum?

A: Mathematics has a subject matter which involves much more than calculation. CSMP believes that the number system is very important and that it is essential to introduce the pupil to the world of numbers. However, the intelligibility of that world is not essentially a matter of computation. The teaching aids of CSMP give concrete support to some important internal relations in the world of numbers as well as some logical reasoning. The deep understanding of mathematics is not always verbal but is very often pictorial and CSMP tries to respect that aspect.

5. Q: Why use the languages of strings, arrows, and the Papy Minicomputer?

A: It is surprising that the language of strings was not used earlier and more extensively in the teaching of mathematics. Mathematicians have used strings for a very long time and they support notions so fundamental and natural that they go back to Aristotle. The scope of the string diagrams is broader than mathematics, the string diagram being a marvelous instrument for logical thinking. String diagrams permit the understanding of a situation at a glance.

Arrow diagrams are probably the most important teaching aid in CSMP. They are used everywhere in our daily lives; signals, advertisements, directions, and so on. Why not use them in teaching mathematics? Arrows are a pictorial way to represent relations. Relations have always been important in mathematics, but in the past seemed to be something rather abstract, elusive, and mysterious. With arrows they become vivid, concrete, and easy to understand.

The Minicomputer allows very young children to become familiar with numbers and operations on them (especially large numbers and the more difficult calculations) before they are able to work with them on paper. Later the Minicomputer is used as a vehicle for posing situations involving mental arithmetic and for examining the workings of the arithmetic operations. It provides the experience necessary so that rather than memorizing an algorithm and applying it to situations, the situations give rise to the algorithm.

6. Q: If the students use the Minicomputer, will they be able to calculate without it?

A: This question suggests that the only purpose of the Minicomputer in the CSMP curriculum is to perform calculations. To be sure, the Minicomputer does enable very young children to deal with "large" numbers and perform relatively difficult calculations long before they can write the paper and pencil algorithms for the calculations. But very soon these experiences with the Minicomputer suggest how the paper and pencil algorithms come about, and from that moment the Minicomputer is no longer regarded as a calculating tool. Instead, for the remainder of the curriculum, the Minicomputer assumes its primary roles: as a source of very interesting problems and games, as an effective vehicle for stimulating mental arithmetic, as a teaching device for introducing more advanced topics such as the negative numbers and the decimal numbers, and, in general, as an alternate representation of numbers.

7. Q: What is the CSMP position on hand-calculators and what role do they play in the program?

A: Hand-calculators should be a part of any mathematics curriculum. As more and more children have hand-calculators, these tools will become incorporated in the elementary and secondary curricula in a natural way with uses very similar to those of the Minicomputer. In some cases they may be used as a tool for mechanical computation where the difficulty of the calculation might turn an otherwise interesting problem into an exercise in drudgery. In other cases they may be used as the context for posing situations leading to the discovery of important mathematical properties or providing opportunities for interesting exercises in mental arithmetic.

Hand-calculators are of particular benefit to "slow" students. Many slow learners have mechanical difficulties rather than problems in thinking about the situation. The hand-calculator alleviates this problem.

Some lessons in the CSMP curriculum require only two calculators, while others suggest that each student have one. Lessons of this latter sort, however, also include suggestions on how they may be modified since not every student may have a hand-calculator. In either situation, students with access to hand-calculators should always be encouraged to bring them to class.

8. Q: Why is there a lack of drill and little emphasis on "skills"?

A: CSMP holds that training for rote skill should never precede understanding. The avowed purpose of drill in arithmetic is to acquire the "skills", that is, the skills of calculation. Certain numerical skills are essential, but, unfortunately for school children, the usual drill techniques for acquiring them are dull, stultifying and sometimes even counterproductive. The CSMP curriculum appears to place little emphasis on "skills" because it does not resort to drill. But as a matter of fact, the essential skills of both mental and written calculation are learned almost inadvertantly in response to the many situations where the ability to calculate is needed in order to respond to the situation: to win the number game, to identify the secret number of the detective story, or to build a number road. Always, calculation is considered as a tool for doing interesting things, rather than a chore done for its own sake. See also the comments on Evaluation, Section 2 (d) of the General Introduction.

9. Q: How do students learn to use mathematics in practical situations?

A: For students of elementary school age, a situation that intrigues them and catches their imagination or their curiosity will certainly motivate them to learn whatever is necessary in order to deal with the situation. But situations that adults consider "practical" very often are outside the realm of interest of school children and do not provide motivation for learning. CSMP has found that students will indeed learn to use mathematics in situations they consider worthy of their attention. As they approach adulthood they will soon consider "practical" situations to be most worthy of their attention, but in the meanwhile, they are more interested in games, fantasy, puzzles, stories, and so on.

10. Q: Why are we teaching probability?

A: Next to arithmetic, probability has the most direct application to life in such areas as economics, meteorology, data processing, insurance, most of the natural sciences, and most of the social sciences. Probability differs from many areas of mathematics in that it is applied directly. One does not need to combine knowledge of probability with any deep theoretical knowledge in order to apply it to "real world" problems. One is exposed to statistics nearly every day of one's life, and the better developed one's intuitive sense of probabilistic situations is, the less likely one is to be "duped" by statistics.

11. Q: How are telling time, money, and the metric system treated?

A: Telling time is not included in the CSMP program. It is our belief that telling time is not a part of mathematics, although it is frequently included in the mathematics curriculum. We do not feel that we could improve on the methods currently used, and, for this reason, we urge you to teach this topic using those methods and materials that have proved successful for you in the past.

The topic of money is first introduced in the CSMP curriculum in lessons involving combinatorics. It is treated again in the Upper Primary and Intermediate Grades as a support for the teaching of the decimal numbers. Should the students need more exposure to this topic, teachers should feel free to develop supplementary materials in order to provide extra practice.

The metric system is introduced in the second and third grades, and provides all standardized units of measure for the Geometry and Measurement strand. We recommend that you do not ask the students to convert from the English to the metric system or vice versa (inches to centimeters, yards to meters, Celsius to Fahrenheit, grams to ounces, and so on). Since the United States has not yet "gone metric", your students will still need to be familiar with the English system. We would suggest that you use whatever materials and methods you have found successful in the past in teaching this topic, but we repeat our recommendation that you not ask the students to convert from one system to the other.

12. Q: What is the role of storybooks and story-workbooks in the program?

A: The storybooks, "Stories by Frederique" are a collection of 24 stories for young people, each of which is presented in the format of a newspaper for children. They are intended to appeal to children in the 5-14 age bracket. The stories are completely independent of each other and need no external explanation. They may be used for individual or collective reading, at home or at school.

The story-workbooks, as their name implies, are a cross between storybook and workbook. They present more problems for the student to solve than do the storybooks, but within a story setting. The story-workbooks comprise the collection called "The CSMP Library".

Each of the stories is portrayed by means of the mathematical languages of arrows, dots, and strings which reinforce the ideas of the stories. The young readers are personally involved in each story. They must answer questions and use their imaginations to invent additional adventures for the characters in the story. They learn that there is room in the make-believe world of mathematics for exercising the aesthetic sense and for expressing emotions.

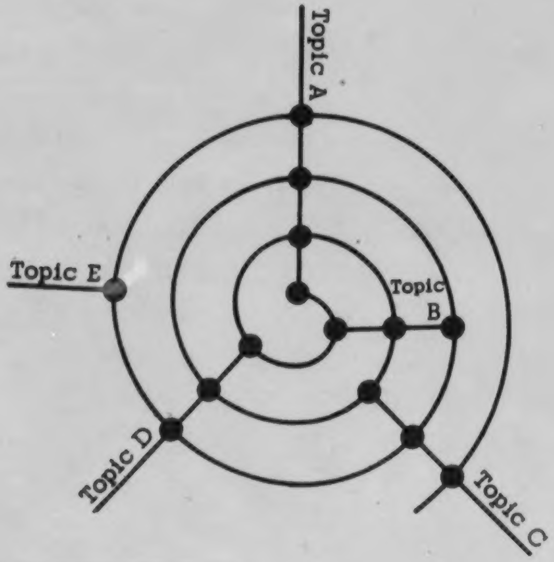
Among the goals of the various storybooks and story-workbooks are the following: to make the children familiar with nonverbal mathematical languages and to leave them with the impression that these languages are as natural and useful as the usual language; to introduce some mathematical games; to present detective stories which introduce some interesting mathematical problems; to provide reinforcement for all students and especially for the slower learners.

13. Q: What should I do if some of the students don't fully understand a lesson?

A: Much of the following appeared in Section 2 (c) but is repeated here for emphasis and your convenience.

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson is an end in itself. It is neither intended nor expected that each child will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations and topics from day to day rather than to continue with one type of situation or piece of content until so-called mastery has occured. Students at this age do not seem to enjoy staying on the same topic for a very long time, but prefer to be involved in a variety of situations. Thus, the lessons are scheduled so that different topics will be studied during each week and similar situations appear again and again at different times and levels in a spiral development.

If a spiral curriculum seems strange at first, consider the following illustration: Imagine an academic subject that could be divided into five subtopics. Then imagine a child's progress being represented by a spiral, starting at the center and traveling outwards in the direction of increasing degrees of maturity and sophistication. In the diagram, an intersection point between the spiral and a topic line indicates a lesson on that topic.



The child is first introduced to Topic A, then to B, then to D, followed by a slightly more sophisticated lesson on A, and so on. Topics C and E are not introduced until a little later in the spiral, while Topic B may be of a more terminal nature and is considered completed after a few lessons.

This spiral development, in which a child experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging, is wholly consistent with CSMP's pedagogy of situations.

Acceptance of the CSMP pedagogy requires a certain act of faith on your part. If you have been accustomed to staying with a given lesson until everybody (or nearly everybody) in the class knows it "cold", then you may be initially very reluctant to leave a lesson at the end of the time suggested, believing that several children "haven't got it yet". By keeping the spiral idea in mind, you may become more comfortable with these time schedules. Although it may appear to you that no learning is taking place in a sequence of lessons, you may be surprised that suddenly even your slower students are understanding the ideas when they encounter them later in the spiral. Whereas with a mastery approach, presented in a purely individualized mode, slow students often linger most of the school year trying to "master" low level skills (with generally poor results), the spiral approach gives these students the opportunity to benefit from the richness of the situations presented throughout the curriculum. In this manner children are allowed to become involved in interesting and enjoyable situations which are modeled by important mathematical notions. They are not deprived of a variety of experiences just because they haven't mastered every bit and piece along the way.

14. Q: Should I stick to this approach even if some students are visibly upset over not having comprehended a lesson?

A: If a child is upset, that child has been made to feel upset, probably by a reversion to the mastery approach. In the spiral approach each situation offers something for each child. The child will only be upset if your attitude indicates that there was more to the situation than the child was able to understand. Every experience in life may be understood at different levels. This is naturally accepted by people except when it is indicated to them that they should have a deeper understanding.

Rather than abandoning the spiral approach, if a child seems upset, stick more closely to it. Find a way for the child to express what understanding he or she does have and accept that expression in a positive manner.

15. Q: What provisions are made for the slow learner?

A: The spiral approach provides for the slow learner by introducing situations that each child can experience and learn from at his or her own level. The next time the spiral reaches a similar situation each child's level of understanding will be somewhat higher but again the experience and learning take place at each child's own level. This is as true for the slow learner as it is for any other student.

The slow learner should participate in math with the other students in the class but might benefit from additional small group work. The additional time is not to rehash or drill on the situations presented in class, but rather to provide additional experiences for these students. See the book "Math Play Therapy" for suggestions for working with slow learners.

16. Q: While providing for the slow learner, how do I challenge the brighter students?

A: The spiral approach is very good for addressing the needs of all levels of students and gives students a chance to excel in mathematics. The situations presented are often open-ended and allow the student to experience and understand them at as high a level as the child is capable.

In order to accommodate the various ability levels in a given class, the worksheets generally are written at several levels of difficulty denoted by stars. "*" represents an easy worksheet, "**" represents an average level worksheet, and "***" and "****" represent difficult level worksheets. All students should begin with the "*" level worksheets for a given lesson or sequence of lessons and proceed as far as possible through the other levels. In classes containing the usual range of abilities, most of the students should succeed with the one-star worksheets; the two-star worksheets should be appropriate for about two-thirds of the class; and approximately one-third of the students will find the three- or four-star worksheets challenging and enjoyable to do. The workbook series are usually written at different levels of difficulty in the same manner as the worksheets.

If your school is flexible, there is also the possibility of letting your students take math with a higher or lower grade level class. For example, your very bright second graders could go to a third or fourth grade class for math instruction.

17. Q: Is it necessary to follow the suggested schedule exactly?

A: This program is not meant to be a static sequence of exercises to be followed rigidly. The lessons in this guide are suggestions and the time allotments are merely approximations based on our own experience teaching the materials. If this will be the first time you are teaching the program and you would like specific suggestions for organizing the lessons, we offer some guidelines and a suggested schedule of lessons. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate.

It is probably best to follow a one-day, one-strand pattern when you organize your own schedule, rather than to spend two or three full weeks "covering" one particular topic. Interchange the strands and the days of the week as you please. Three days each week should be devoted to the two parts of the World of Numbers strand and the Workbook strand.

Exercise your own professional judgment in deciding when to stop one particular lesson and when to take it up again. Just be sure that you do not insist that every single child understands every single aspect of a lesson before you go on. We suggest that you make a note of where you end a given lesson so that you will know where to begin the next time you teach it.

18. Q: How do I handle individual students who are new to CSMP?

A: In most cases, the differences between new and veteran students tend to fade rather quickly. One exception might be in lessons involving the Minicomputer. Veteran students are usually adept with the Minicomputer and often are very good tutors for the new students in the use of this abacus. In addition, the storybook "Two By Two" is suggested as an aid for new students who are unfamiliar with the Minicomputer. Some of the storybooks and story-workbooks written for younger children can be used to introduce new students to the languages of strings and arrows.

19. Q: How do I handle whole classes who are new to CSMP?

A: CSMP does not assume that Kindergarten and first grade students have any prior experience with CSMP. The Kindergarten and First Grade Programs will need no adaptation for classes who are new to CSMP.

Second grade classes which are new to CSMP will need to be introduced to the CSMP languages. A suggested adaptation of the second grade program is included in the CSMP Mathematics for the Upper Primary Grades (UPG) Part I teacher's guide.

Third grade classes which are new to CSMP will need to become fluent in the CSMP languages before beginning the lessons in UPG Part III. A special schedule has been prepared for these classes which consists of prerequisite lessons and workbooks from UPG Part I and II followed by a modified UPG Part III and IV program.

Fourth grade classes which are new to CSMP will begin with CSMP Mathematics for the Intermediate Grades, Entry Module I which lasts one semester, before the class begins the regular fourth grade program. Future development plans provide for the creation of Entry Modules II and III for entry into the CSMP curriculum at the fifth and sixth grades, respectively.

20. Q: What should I do with my class if they did not finish the previous year's schedule?

A: No special measures are required. Each level of the CSMP curriculum begins with introductory lessons. In addition, the spiral approach to the lessons permits each student to participate at his or her own level of understanding, even if that level is not quite as high as it might have been if the previous year's schedule had been completed.

21. Q: How do I handle a student who has an extended illness?

A: The spiral approach used by CSMP permits a student to miss one or two weeks of school without needing much catch up work. When the student returns to school either you or one of his or her classmates can assist him or her with any material for which some help is deemed necessary.

If a student misses several months of classes and experiences difficulty with any of the CSMP languages upon return, he or she can be given the same help as you would give a transfer student new to CSMP.

Because of the nature of the materials and the parents' possible unfamiliarity with the languages, it is probably best not to have the worksheets and/or workbooks sent home to a sick child. In fact, without the lesson or collective introduction which often precedes the individual work on worksheets and workbooks, it would be practically impossible for the student to proceed at all.

22. Q: How can I assess student progress and assign grades?

A: CSMP does not believe that it is either desirable or reasonable for teachers to attempt to evaluate student mastery of skills from day to day or even from week to week. On the other hand, teachers, parents and even the children themselves, benefit by some monitoring procedure that records involvement in activities and assesses progress through the program.

Most teachers have their own monitoring devices with which they are comfortable. It is recommended that such devices be continued.

23. Q: How do CSMP students do on standardized tests?

A: While CSMP is opposed to the philosophy underlying standardized testing, test results are available to us from several sources. The results indicate that CSMP classes generally do as well or better on tests than do non-CSMP classes. It should be pointed out that standardized testing and traditional programs are correlated and that the CSMP student has been exposed to a great deal of mathematics that is not taught in more traditional programs or considered in the construction of standardized tests. There are also many immeasurable or unmeasured areas such as the enjoyment of mathematics, strategic thinking, organizational ability and depth of understanding of the methods, power and limits of mathematics where we feel CSMP students have a definite advantage. See also the comments in Evaluation, Section 2 (d) of the General Introduction.

24. Q: What problems would CSMP students have, if they return to a more traditional program in the next school year?

A: There should be no problems for CSMP students who return to a traditional program. Test results indicate that CSMP students are at or above the level of non-CSMP students in material covered in traditional courses. In addition, CSMP students have encountered many situations that are not introduced in traditional programs nor tested for on standardized tests. (See Question 23) In particular, their reasoning ability should be much sharper than their non-CSMP counterparts.

There are, of course, differences in timing as to when different programs introduce the standard algorithms and often CSMP's introduction will occur later than in some of the other programs. The reason for this is our belief that it is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

CSMP students can usually solve a problem before having learned the standard algorithm. If they enter a program that expects an algorithm they have not learned, they are still more than adequately prepared for the algorithm and should have no trouble quickly learning to use it.

CSMP introduces the usual addition algorithm in the second grade, one of the two standard subtraction algorithms in the third grade, the multiplication algorithm in the fourth grade, and the division algorithm in the fifth grade.

25. Q: Why doesn't each student receive every workbook?

A: The students are of various ability levels in every class. Some students will be able to work only on "*" and others on "*" and "**" but not on "***" or "****" problems, so there is no need to give a complete set of workbooks to every student. (See Question 16) For this reason the usual assortment for a class of 30 students contains 30 of each "*" and "**" workbook (one for each student), and 10 copies of the "***" workbooks. If we were to include one copy of each workbook for every student, many would be wasted and the cost of the program would increase.

26. Q: How can I explain CSMP to parents?

A: You will find a letter to parents at the end of this section which may be reproduced and sent home with the students.

The best way would be for your coordinator to organize a workshop for interested parents. The parents can then see the languages of the program in use and have some of the same experiences their children have had. Workshops can be of almost any length but anything short of two hours does not allow for much familiarity with the material. In such a workshop we would suggest that the language of the Minicomputer be explained with particular emphasis on its use in presenting situations where mental arithmetic is called upon and in the development of algorithms (for example, physically carrying a checker from the ones' to the tens' board in the "8 + 2 = 10" play, paralleling "carrying" in the paper and pencil operation). A lesson using the storybook "Where's My Nose?" provides a good example of the power of the language of strings. A nonnumerical example, such as the brother-sister graph found at various grade levels, and a numerical example, such as building a road from 5 to 55 using +3 and 5 × arrows, will provide exposure to the language of arrows.

27. Q: What are some ways to organize materials and manage my class?

A: It is very important to organize your CSMP materials before school begins. Decide where you will store the materials once they are organized. Filing drawers, shelves, boxes, or closets are all satisfactory as long as you can easily get to the materials. Take them all out of their shipping boxes and spread them on several tables and/or desk tops in your classroom so they can be organized into the order you will be using them.

Workbooks: The teacher's guide (or guides) are needed from second grade onward to find the order in which the workbooks are used. Find the table of contents in the Workbook strand. Collect all the workbooks in the order which they are listed. Some workbooks have identical titles with only the number of stars in the upper right-hand corner differing. Always gather the one- and two-star level workbooks first, and then the three-star workbooks of a particular title. In the case of the story-workbooks, only one level exists. In this case, ignore the statement above regarding three-star and/or two-star workbooks respectively.

After organizing all the workbooks, place them in an upright position in your file drawer, box, shelf, etc., so that the first workbook to be used is at the front of the stack.

Worksheets: This grade level has a separate worksheet booklet for each student and you should decide whether you will store these booklets and pass them out when a particular lesson calls for a worksheet, or simply let the students keep their own worksheet booklets in their desks.

Each worksheet can be removed from the booklet or left in depending on how you wish to manage worksheets. However, once a worksheet is removed from the booklet it cannot be replaced, so make sure students are careful about tearing off the correct sheet.

Student Manipulatives: Storage of each manipulative in a box, properly marked, is the best way to make everything accessible; a shoe box for the compasses and translators, an envelope for the spinners, and so on.

Individual Minicomputer sheets can be better preserved if you use a clear contact paper over the top. This will prevent bending, etc., of the sheets.

Additional Materials: Be sure that you have plenty of high quality colored chalk, large sheets of unlined white paper, and that your students have colored pencils, pens, or crayons. Plan well ahead to be sure you have any needed materials that are not supplied by CSMP.

Keep all teacher materials where they are easily accessible. You may find the guides easier to use if they are put in looseleaf binders. The demonstration Minicomputer boards and the number line should always be on display in the room.

28. Q: Much of the CSMP material is new to me. How do I learn enough to teach a class?

A: Every CSMP teacher should have been trained by his or her coordinator. Kindergarten teachers receive eight hours training, first grade teachers sixteen hours, second grade teachers twenty-four hours, third grade teachers thirty-two hours, and fourth grade teachers forty hours. During the training sessions the CSMP content is taught to new teachers and suggestions are given for organization of their classes.

If the suggested training schedule has been followed, many lessons taken from the CSMP curriculum, or very similar lessons, have been presented. The schedule also makes use of the spiral approach so that training sessions aim not only to teach CSMP mathematics but to familiarize teachers with the curriculum and pedagogy of CSMP.

29. Q: What should be done when there is a substitute teacher?

A: Some school systems have included in their CSMP training program people who frequently substitute. When this is not the case, it would be very difficult for the untrained substitute to teach from CSMP materials. One solution is to arrange for another CSMP teacher to teach math to your class while the substitute teacher teaches another subject to that teacher's class. If that can't be arranged and you will only miss one or two days, you may find certain CSMP material, uncompleted workbooks for example, that the substitute can manage. Otherwise, use those days to cover material not included in the CSMP curriculum, such as telling time. If your absence will be an extended one, arrangements should be made with your coordinator to train a substitute, rearrange schedules to put a CSMP teacher in your class, or otherwise provide for your students.

30. Q: The program makes use of the student's creative freedom. How much room is there for the teacher's ingenuity?

A: CSMP not only allows for your ingenuity but depends on it. While the lessons may appear to be very rigid by presenting a dialogue for teacher and student, it should be remembered that these are suggested questions, answers, and comments based on our experience with development classes. Even if you were to follow your part of the "script" your students are likely to offer responses which differ from those we have listed. It is the student's response that will determine the content and direction of your next question or comment.

In a traditional program your creativity is called upon the first time you develop a particular lesson, and to a lesser extent as you revise lessons in succeeding years. There is little room for spur-of-the-moment creativity once a lesson begins. The CSMP curriculum, by creating a free and open situation emphasizing the student's creative freedom, continually presents you with unanticipated student responses and calls on your ingenuity at all times.

TO THE PARENTS OF CSMP STUDENTS

Your child is participating in the Comprehensive School Mathematics Program's (CSMP) elementary school curriculum, developed under a contract with the National Institute of Education. While the program cannot be fully explained in so brief a format, it is hoped that this letter will give you some feeling for the substance of your child's mathematics education.

The content goals of the CSMP K-6 program include a development of the standard number systems and their operations (including an early introduction to the integers, rational numbers and decimal numbers) interwoven with a two-pronged development of geometry (a rich variety of experiences in combinatorial and metric geometry together with a carefully developed sequence of activities in parallelogram (affine) geometry). On an equal footing with the above, the CSMP program includes a strong component of probability, statistics, and combinatorics which extends throughout grades 1-6. Powerful pedagogical devices are exploited providing several nonverbal languages which render the unifying mathematical ideas underlying the content accessible to very young children. Through these languages, children are able to apply mathematical ideas effectively and creatively to explore the content directly, unemcumbered by the terminology and notation which usually pervade the early stages of the curriculum and stand as a barrier between the children and the content. These languages include:

1) The Language of Strings This "language" mimics the basic methods of collecting and classifying data relevant to the problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of set.

2) The Language of Arrows This graphical language models the fundamental process involved in comparing and analyzing sets and operations on them. That is, it deals with the mathematical notions of relations and functions including, but not restricted to, ordering relations and the numerical processes of adding to, subtracting from, multiplying by, and sharing equally among.

3) The Language of the Papy Minicomputer The Papy Minicomputer, an extremely effective paper abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to a multitude of numerical algorithms, including the standard algorithms for the basic numerical operations.

Among other things, these languages allow for the development of the basic operations on numbers in a natural way based on previous experiences with the languages. They also provide a context in which situations arise that call for the repeated use of arithmetic operations without resorting to drill exercises, which most children find boring.

Another tool, not unique to but used by CSMP, is the hand-calculator. The hand-calculator allows the child to study problems that call for calculation beyond the child's ability or in which the calculation is so extensive as to cause the child to lose interest. The calculator is also used in a way similar to the use of languages mentioned above. Properly used, it can help lead to the development of important mathematical ideas and present situations for the interesting use of mental or paper and pencil arithmetic.

The content is learned in an atmosphere of constant applications utilizing a "pedagogy of situations".* These vary from simple story situations in the early primary grades to challenging applications and nontrivial simulations of real world problems in the upper grades.

One of the tenets of CSMP's development philosophy is that mathematics is a unified whole and should be learned as such. As a result, along with CSMP's concern with the variation in ability levels in the typical heterogeneous class, the curriculum is structured in a spiral format throughout the K-6 program. The content is completely sequenced in spiral form so that each student is brought into contact with each area of content continuously throughout the program. This consciously precludes atomizing the content and mastering each bit before continuing to the next. Rather, each student spirals through repeated exposures to the content, building interlocking experiences of increasing sophistication.

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* Inspired by the work of the mathematician and psychologist, Caleb Gattegno.

Another tenet of CSMP's development philosophy, based on years of experimentation in the classroom, is that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation, and individual independent experiences. Individualization throughout the K-6 program is achieved through graded series of workbooks integrated into the basic sequence of lessons. Due to the spiral nature of the curriculum, each student can engage in a workbook series without losing touch with the progress of the class while at the same time working his or her way up through the series to the limits of his or her ability.

A feature unique to the CSMP program is a collection of storybooks written by Frederique Papy (STORIES BY FREDERIQUE) and a collection of story-workbooks (THE CSMP LIBRARY). These probe the affective domain of learning and bring children into contact with mathematics at levels until now unexplored.

CSMP does not have its own system of student assessment and progress reporting. While standardized testing indicates that CSMP students develop the "basic skills" as early or earlier than non-CSMP students, we hope you will not be obsessed with your child's development of these "skills". Each child is an individual and should not be judged against norms but on what he or she can and does accomplish. Many extremely important areas --- development of self-confidence, ability to think and reason, etc. --- are very difficult to measure and grade. If you have any questions regarding the progress of your child, his or her teacher is in the best position to answer those questions.

THE CSMP STAFF

WORKBOOKS

Comprehensive School Mathematics Program

Mathematics for the Intermediate Grades
Part II

Workbooks

CEMREL, Inc.

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WORKBOOKS

INTRODUCTION

There are many opportunities for the students to work individually during the course of the lessons described in the four content strands. In the workbook strand, however, it is this individualized work which becomes the chief end of the lessons. It has been our goal in this strand to provide the students with opportunities to

- review many of the ideas they have met in the content strands
- apply their acquired knowledge to new situations requiring various kinds of strategic thinking
- learn how to read and use mathematics workbooks and story-workbooks.

The following workbooks are provided.

A Set of Problems No. 1
A Set of Problems No. 2
A Set of Problems No. 3

The following story-workbooks are provided.

Clinton Street (Books A and B)
A Strange Country

One lesson is based on the storybook, NABU WINS AN AWARD. The lesson titled "Arrow Roads" does not use either a storybook or a workbook, but is included in this strand because of the support it gives to the "Clinton Street" story-workbook. The three workbook series, A Set of Problems - Nos. 1, 2 and 3 are written at three levels of difficulty. These levels of difficulty are coded as:

* easy; ** average; *** challenging.

We suggest that you devote one day a week to this strand. We provide a complete suggested schedule for "W" lessons immediately following this introduction. It is suggested that you follow this schedule more or less as given, even if many of your students have not finished a particular workbook after the two or three days that are allotted to it. As a general rule, students at this age do not enjoy remaining on the same topic for too long. And besides, it is a basic principle of the CSMP spiral approach that a variety of experiences is preferable to any attempt to master a given topic by continued "plugging away" at it. This principle applies as much (if not more so) to the workbook strand as it does elsewhere in the curriculum. If you do feel, however, that more time is needed for a particular workbook, you may find it convenient to continue work on that workbook during one (or more) of the adjustment days that occur in the suggested schedules for the four content strands.

For each of the story-workbooks dealt with in this part of the Teacher's Guide, there are collective lessons that accompany the story-workbook. These are followed by an answer key for that story-workbook. There are also some short collective (warmup) lessons suggested for two of the three workbook series. Following these suggestions there are answer keys for the workbook series. All answer keys are presented in reduced format, each page actually containing reduced copies of four of the pages of the original booklet, with solutions written in.

For each workbook series we suggest that all students start work at the easiest level of difficulty and then proceed as far as they can through the other levels in the time that is to be devoted to that series. You can expect that, under normal circumstances, about two-thirds of the students will work through the second level of difficulty, and about one-third will reach the most challenging level. These assumptions are reflected in the fact that a full classroom set of materials contains 30 copies of each of the * and ** workbooks (* and ** are combined into one booklet) and only 10 copies of the *** workbooks. Additional copies of these workbooks (all levels) can be ordered as separate items on the order form for Part II of the Intermediate Grades Program.

The workbooks provide an excellent instrument with which you can assess the progress of your students on a week-by-week basis. To use this instrument effectively certainly does not require that you check every page and every problem for each student. You should develop the sort of procedure for checking students' work with which you can feel most comfortable. This might take the form of checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or simply looking carefully at the first few pages to be sure the students have understood the general idea of the problems in a particular book. In our experience, an effective procedure is to clip to the front cover of each checked workbook a strip of paper on which can be indicated your assessment of the student's work on each page you check. For this purpose, you might use the following symbols.

meaning "well done" or "correct"

meaning "something wrong, try again"

meaning "did you forget this problem" or "do you need a hint?"

On the next page we show the front cover of one student's A Set of Problems No. 1 (* and **) workbook after it has been checked.

Immediately following the suggested schedule for "W" lessons we provide a progress chart to help you keep track of

- where each of your students is on any particular workbook day
- how many levels each student has completed under each title
- your personal degree of satisfaction with the student's work on each series.

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Name Beverly W.

* and **

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32 ___

(Note: This is the way the paper was returned to Beverly at the beginning of the second lesson devoted to this workbook series. The blanks on lines 22-32 indicate that she had not yet worked on these problems. Before Beverly begins on these pages during the second lesson, she should correct her mistakes on page 11 and finish her work on pages 4, 7, 13, 14 and 20.

There are no problems on page 17; this page is the separation page between the * and ** levels of this workbook series.)

In particular you should be on the watch to see if for any of your students even the easiest workbooks are too difficult. Such students would probably benefit a great deal if you were able to devote some time with them to using some of the suggestions made in the book "Math Play Therapy".

We now provide a suggested schedule for "W" lessons. The placement of the various workbooks has been determined by bearing in mind what knowledge is assumed and where the lessons that deal with that information occur in the suggested schedule for the four content strands. Hence, you are warned to be particularly careful when considering possible modification of this suggested schedule. Whatever modification you might like to make, we strongly suggest that you use one day each week for the workbooks.

Here is a list of some of the important points to bear in mind when working in this strand.

- All students begin at the easiest level of each workbook series and progress as far as they can.
- All students begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students are expected to complete all levels of a given workbook series. Only the faster students will reach the most challenging level. The slower students may succeed only at the easiest level, although you should not assume that this will automatically be the case--surprises are not at all uncommon in this area.
- Read the introductory material for each workbook series and story-workbook and give the short introductory collective lesson (s).

SUGGESTED SCHEDULE FOR W – LESSONS

WEEK	Workbooks	Materials
1	**1W** A Set of Problems No. 1 p. 2	Teacher: Minicomputer set*; colored chalk Student: Paper; colored pencils; "A Set of Problems No. 1"
2	**2W** A Set of Problems No. 1 p. 7	T: Colored chalk S: "A Set of Problems No. 1"
3	**3W** A Set of Problems No. 1 p. 11	T: Colored chalk; Minicomputer set S: "A Set of Problems No. 1"
4	**4W** Arrow Roads p. 27	T: Colored chalk S: Colored pencils; Worksheets N97, N98, N99, N100
5	**5W** A Set of Problems No. 2 p. 36	T: None S: Colored pencils; "A Set of Problems No. 2"
6	**6W** A Set of Problems No. 2 p. 37	T: Colored chalk S: Colored pencils; "A Set of Problems No. 2"
7	**7W** A Set of Problems No. 2 p. 40	T: Colored chalk S: Colored pencils; "A Set of Problems No. 2"
8	**8W** Clinton Street p. 55	T: Minicomputer set S: "Clinton Street"; Worksheets N101, N102, N103
9	**9W** Clinton Street p. 60	T: Colored chalk S: "Clinton Street"; Worksheet N104

* A teacher's Minicomputer set consists of four demonstration Minicomputer boards and a sufficient number of demonstration Minicomputer checkers. A student's Minicomputer set consists of two individual Minicomputer sheets (two boards per sheet) and a card of individual checkers.

SUGGESTED SCHEDULE FOR W – LESSONS

WEEK	Workbooks	Materials
10	**10W** Clinton Street p. 66	T: Minicomputer set; colored chalk S: Colored pencils; "Clinton Street"
11	**11W** Clinton Street p. 71	T: Colored chalk S: "Clinton Street"; Worksheets N105, N106, N107
12	**12W** A Set of Problems No. 3 p. 85	T: None S: Colored pencils; 30 cm ruler; "A Set of Problems No. 3"
13	**13W** A Set of Problems No. 3 p. 85	T: None S: Colored pencils; 30 cm ruler; "A Set of Problems No. 3"
14	**14W** A Set of Problems No. 3 p. 85	T: None S: Colored pencils; 30 cm ruler; "A Set of Problems No. 3"
15	**15W** Nabu Wins An Award p. 99	T: "Nabu Wins An Award" S: "Nabu Wins An Award
16	**16W** A Strange Country p. 103	T: Colored chalk S: Large sheets of unlined paper; colored pencils; "A Strange Country"
17	**17W** A Strange Country p. 109	T: Colored chalk S: Colored pencils; "A Strange Country"
18	**18W** A Strange Country p. 113	T: Colored chalk S: Large sheets of unlined paper; colored pencils; "A Strange Country"

Workbook Progress Chart

	A Set of Problems No. 1		A Set of Problems No. 2	
Name	* and **	***	* and **	***

Workbook Progress Chart

	Clinton Street		A Set of Problems No. 3		A Strange Country
Name	A	B	* and **	***	

A SET OF PROBLEMS NO. 1

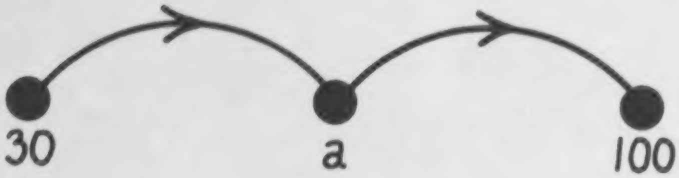
A SET OF PROBLEMS NO. 1

This workbook series has three levels of difficulty (* and ** in one booklet and *** in another booklet). It is suggested that all the students begin with level * and continue through as many levels as they can handle during the three lessons scheduled for this series. Not all students are expected to complete all three levels of this workbook series. Only the faster students will reach the most challenging level (***).

Lesson 1

Exercise 1

Draw this arrow picture on the board.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

T: Suppose the blue arrow is for - 10. What number is here (point to a)?

S: 20.

T: What could the red arrow be for?

S: + 80, because 20 + 80 = 100.

S: x5, because 20 x 5 = 100.

Note: There are other possibilities, but these are the most natural responses.

T: Let's consider a different situation. Suppose the red arrow is for - 20. Then what number is here (point to a)?

S: 120.

It is very likely that a student will answer 80. Should this happen, point to a, trace the red arrow and ask: Is 80 - 20 = 100? (No) When 120 is suggested, be sure to check it by pointing to a, tracing the red arrow, and asking: Is 120 - 20 = 100? (Yes)

T: What could the blue arrow be for?

S: ×4, because 30 × 4 = 120.

S: +90, because 30 + 90 = 120.

Other answers are possible, but these are the most likely responses.

T: Now, let's suppose the red arrow is for ÷3. What number would be here (point to a)?

S: 300.

Trace a return arrow from 100 to a.

T: What could this return arrow be for?

S: ×3.

T: What could the blue arrow be for?

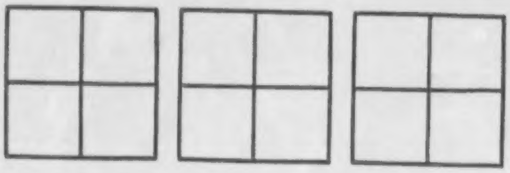
S: ×10, because 30 × 10 = 300.

S: + 270, because 30 + 270 = 300.

These are only the most likely responses. Other answers could be correct.

Exercise 2

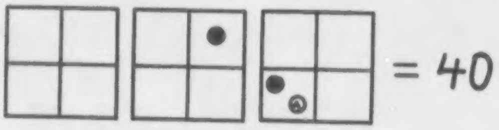
Display three Minicomputer boards.

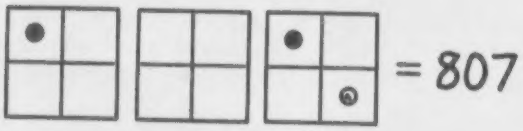


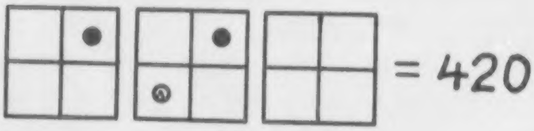
T: I would like someone to put any number he or she wants on the Minicomputer using exactly two positive checkers and one negative checker. The rest of you will write the number on a piece of paper.

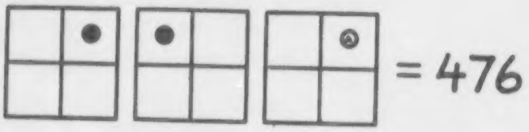
Give several students an opportunity to put numbers on the Minicomputer in this way.

It is difficult to predict the reactions of the students because there are so many possible answers. You will probably be surprised by the variety of their suggestions. Below are some responses that were made in our development classes.

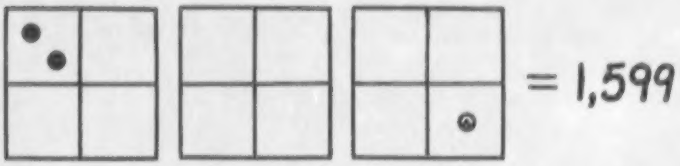






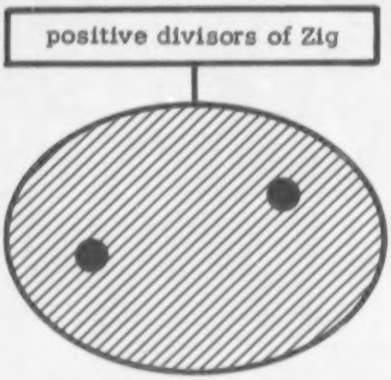


T: What is the largest number we can put on the Minicomputer using exactly two positive checkers and one negative checker?



Exercise 3

Draw this string picture on the board.

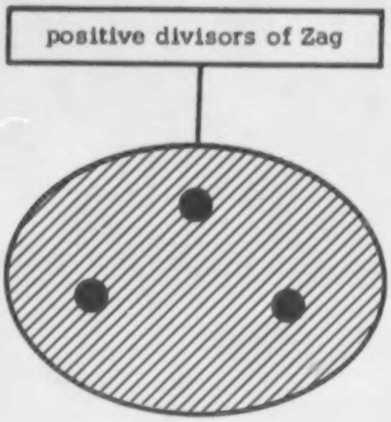


T: On your paper, make a list of as many numbers as you can that Zig could be.

Let the students explore this situation. Encourage students to find several numbers that Zig could be. After a while, make a list at the board from student suggestions of numbers that Zig could be. Accept any prime number for Zig. It may be necessary to stop and count divisors of a number to be sure that it has exactly two positive divisors.

1W

Now draw this string picture.



T: Make a list of as many numbers as you can that Zag could be.

Encourage the students to find several numbers that Zag could be. After a while, make a list at the board from student suggestions. Accept any square of a prime number (e.g., 4, 9, 25, 49, 121, etc.) as correct. It is likely that someone in your class will notice the squaring pattern and suggest that Zag could be 16 or 36. A quick count of positive divisors of 16 and 36 will show that neither can be Zag.

For the remainder of the period, give each student a copy of the workbook "A Set of Problems - No. 1" levels * and **, and let them work on their own.

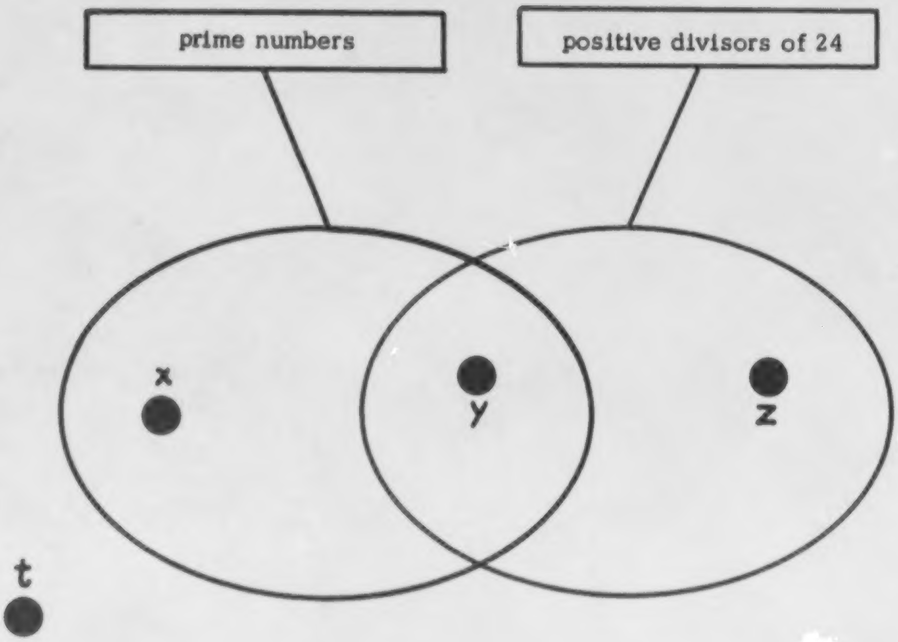
If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of the lesson, collect the workbooks and have them ready to return for Lesson 2.

Lesson 2

Exercise 1

Draw this string picture on the board.



T: What are some numbers that x could be?

Let your students enjoy this open activity. This exercise gives the slower students a wonderful opportunity to participate at their own level. Many answers are possible and you will probably be surprised by the variety of responses.

Answer: Any prime number that is not a divisor of 24, e.g., 5; 7; 23; 11; etc.

Proceed in a similar manner for y, z and t.

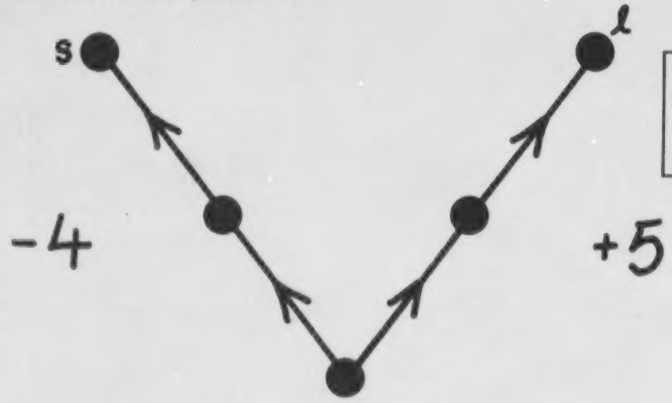
Answers: y can only be 2 or 3.

z can be 1; 4; 6; 8; 12 or 24.

t can be anything that is neither a prime number nor a positive divisor of 24.

Exercise 2

Draw this arrow picture.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

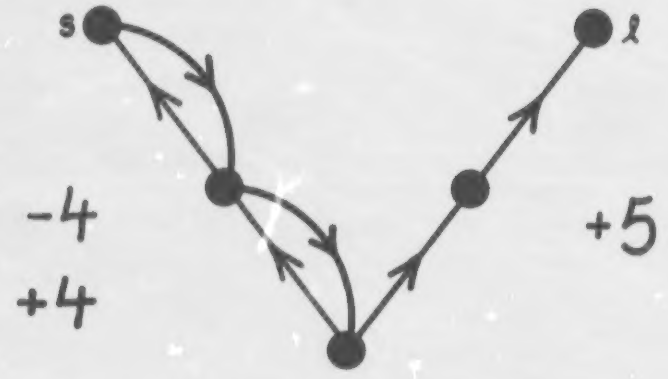
T: Where is the smallest number in this picture?

Let students come to the board and point to the dot that they think is for the smallest number. Suppose a student points to s, the dot for the smallest number.

T: Why is that the smallest?

S: I used the return arrow for -4; +4.

Draw the return arrows for - 4.



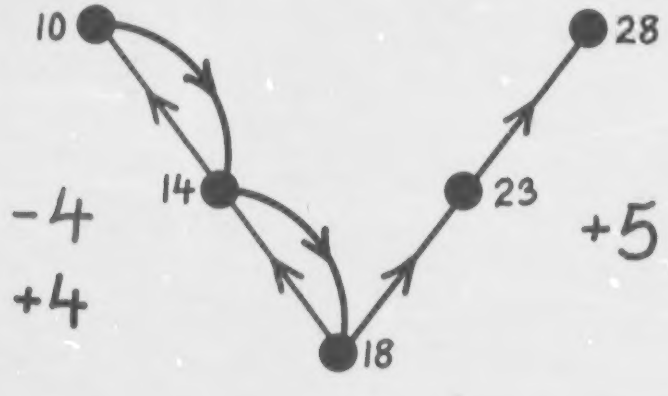
S (pointing to s): All of the numbers will be larger than the number here.

T: Where is the largest number in this picture?

Let a student go to the board and point to it. The dot labeled "ℓ" in the drawing above is for the largest number.

T: Suppose 10 is the smallest number in this picture. Let us put 10 in the picture where it belongs; then label all the other dots and find what the largest number is.

The class should conclude that 28 is the largest number. The completed arrow picture is presented below.



2W

Return the students' copies of "A Set of Problems - No. 1" and ask them to first correct their mistakes (if there are any), then to continue working in their workbooks.

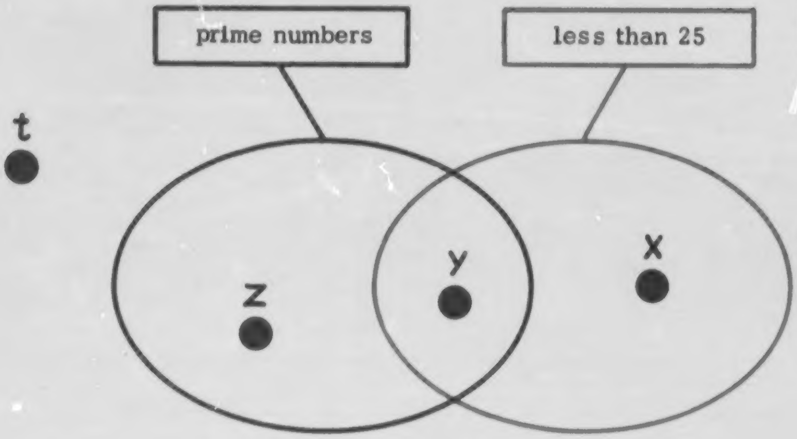
If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem. If a student completes a workbook, give him or her a copy of the next level workbook.

At the end of the lesson, collect the workbooks and have them ready to return for Lesson 3.

Lesson 3

Exercise 1

Draw this string picture but only put the dot labeled "x" in at first. The others will be added later - one at a time.



T: What are some numbers that x could be?

Let your students enjoy this open activity. This exercise gives them a wonderful opportunity to participate at their own level. Many answers are correct; however, you need not insist that all possibilities are suggested.

Answer: Any number that is less than 25 and is not a prime, e.g., 24; 8; 15; etc.

Add the dot labeled "y" and proceed in a similar manner as above. Answer: y can be any prime number less than 25; i.e., 2; 3; 5; 7; 11; 13; 17; 19; or 23.

Continue with the dot labeled "z" in a similar way.

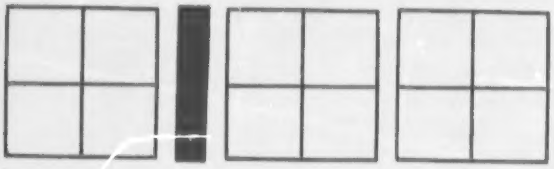
Answer: z can be any prime number greater than 25, e.g., 29; 31; 37; 43; etc.

Finally, the dot labeled "t" has many possibilities. Let those students who have not been participating have an opportunity here. The open-endedness gives everyone a chance to participate.

Answer: t can be any number greater than or equal to 25 that is not prime, e.g., 25; 36; 102; 49; etc.

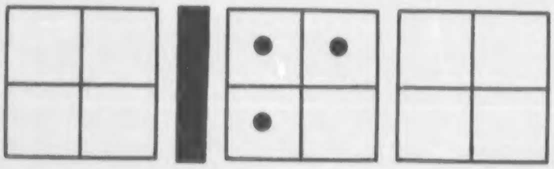
Exercise 2

Display three Minicomputer boards as shown below.



T: I will put a number on the Minicomputer, and you write my number on your paper.

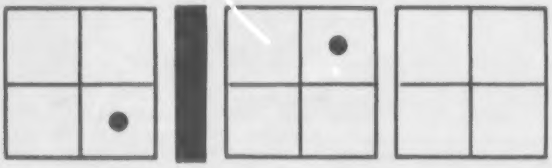
Put the following configuration on the Minicomputer and let the students decode the number.



$= 1.4 = 1.40$

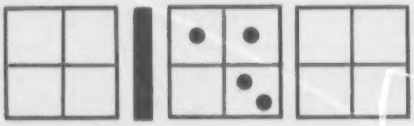
T: Who can write the same number but with one checker less?

Have a student make the move.

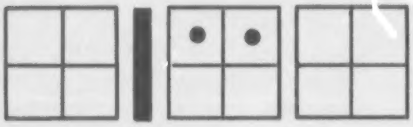


= 1.4 = 1.40

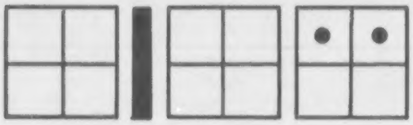
Ask the students to decode the following numbers.



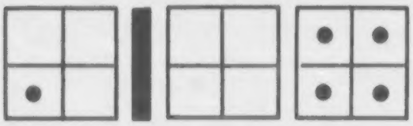
= 1.4 = 1.40



= 1.2 = 1.20



= 0.12



= 2.15

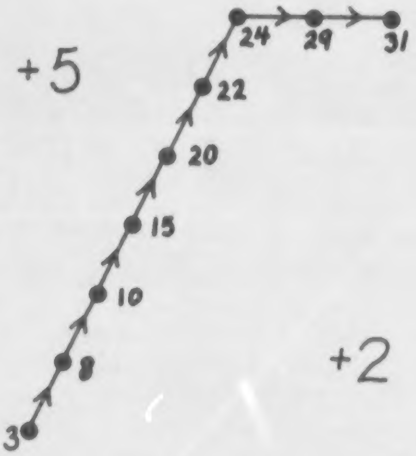
If the above problems give difficulty, encourage thinking about money.

Return the copies of "A Set of Problems - No. 1" and ask the students to correct their mistakes (if there are any), then to continue working in their workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem. If a student finishes a workbook, give him or her a copy of the next level workbook.

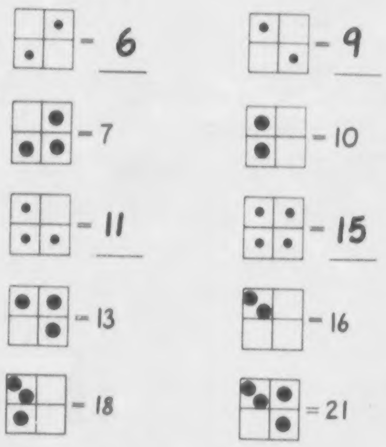
A Set of Problems No. 1 * and **

Which is the largest number in this picture? 31
Label all the dots.



2

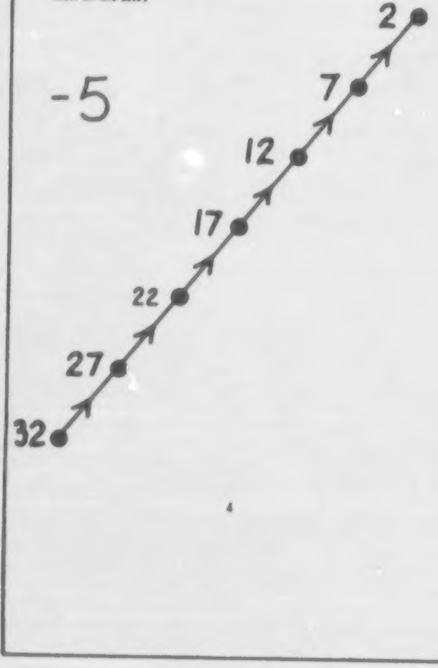
Complete :



3

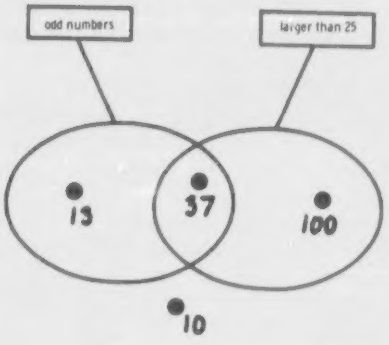
(Other solutions for some of these are possible.)

Ty is the smallest odd number in this picture.
Who is Ty? 7
Label all the dots.



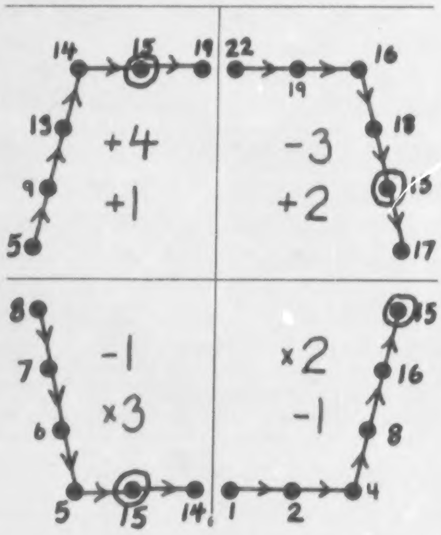
4

Put the numbers 10; 13; 37 and 100 in their correct places in this picture

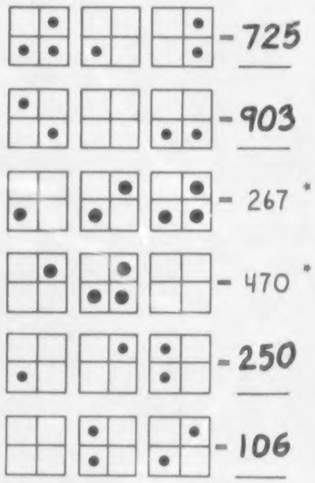


5

15 is in each of these pictures. In each one circle its dot.
Label all the dots.



Complete:

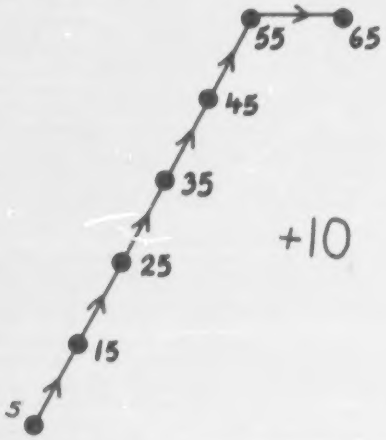


7

*(Other solutions are possible for only these two.)

First clue

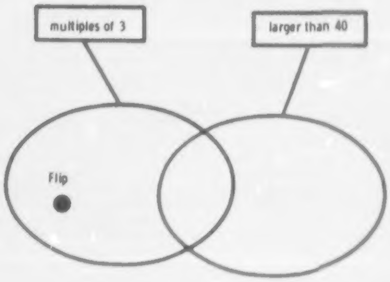
Flip is in this arrow picture.



8

Second clue

Flip is in this string picture.



Flip is 15.

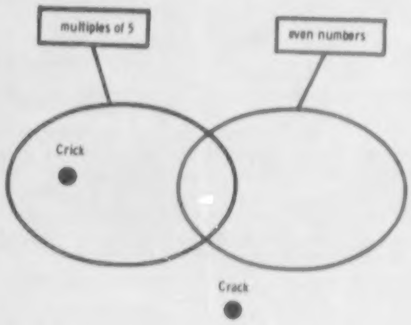
9

Crick and Crack are two of these numbers :

18; 12; 25; 50; 9

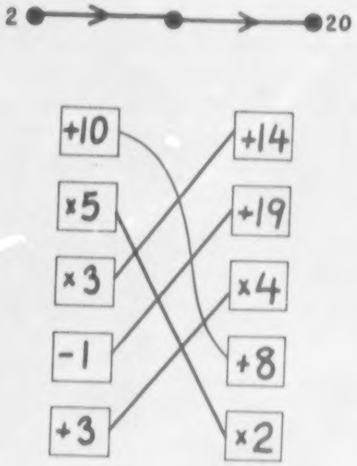
Who is Crick? 25

Who is Crack? 9



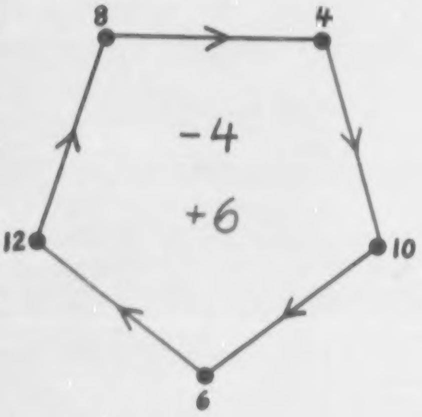
10

Pair the tags. One is done for you.



11

Label the dots as you wish, but be sure your final picture is correct.



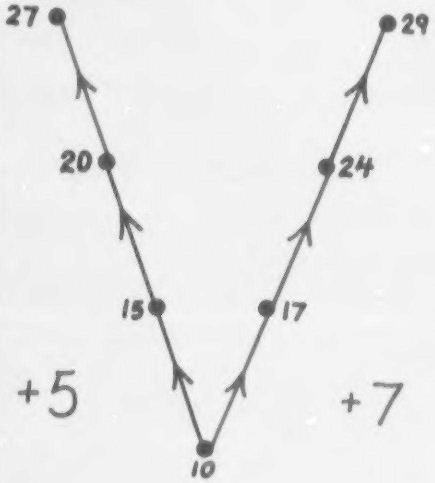
12

(Many solutions are possible.)

10 is the smallest number in this picture and Zip is the largest number.

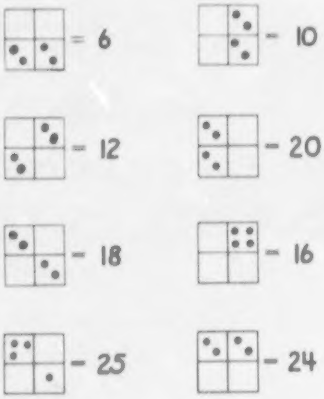
Who is Zip? 29

Label all the dots.



13

Put each of these numbers on the Minicomputer with exactly four checkers.



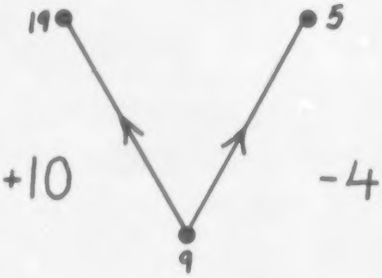
14

(Some of these have several solutions.)

5 is the smallest number in this picture and Rick is the largest number.

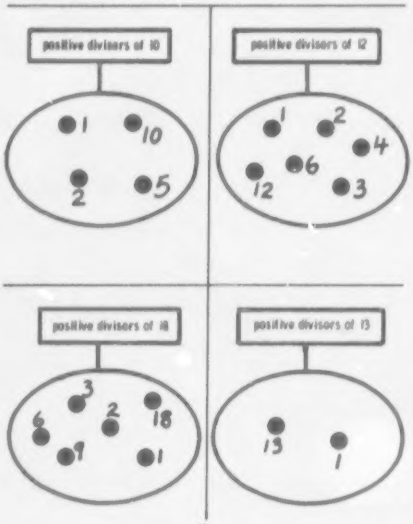
Who is Rick? 19

Label the three dots.

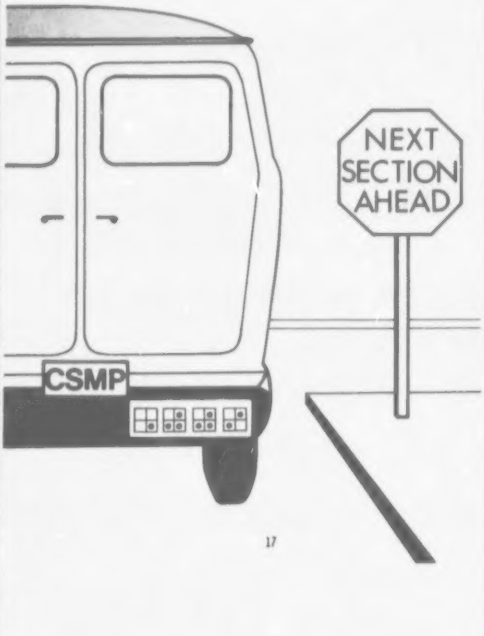


15

Label the dots.



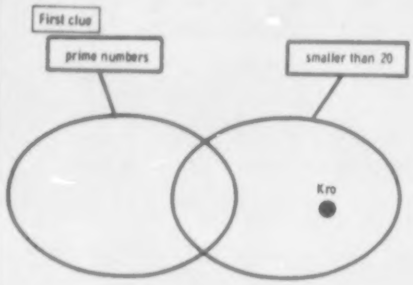
16



17

Kro is a secret number (positive integer).

First clue



Kro could be 1; 4; 6; 8; 9; 10; 12; 14; 15; 16; 18.

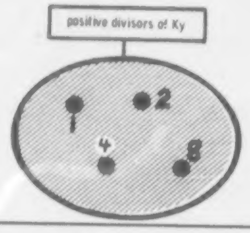
Second clue

Kro cannot be put on the Minicomputer using exactly two checkers (positive or negative).

Kro is 15.

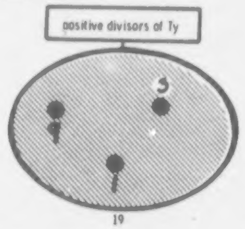
Ky is a secret number.

Who is Ky? 8



Ty is a secret number.

Who is Ty? 9

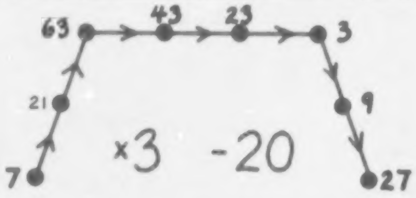


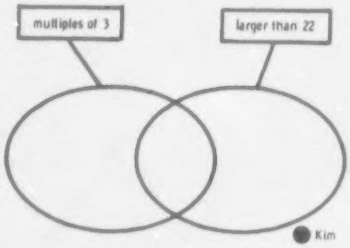
The dots in these two problems can be labeled in different orders, but the same numbers are always involved.

Kim is a secret number.

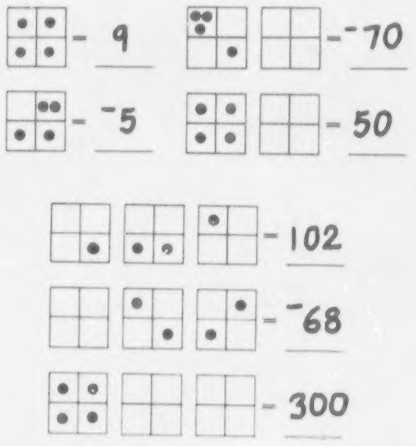
Kim is in this string picture and in this arrow picture.

Who is Kim? 7



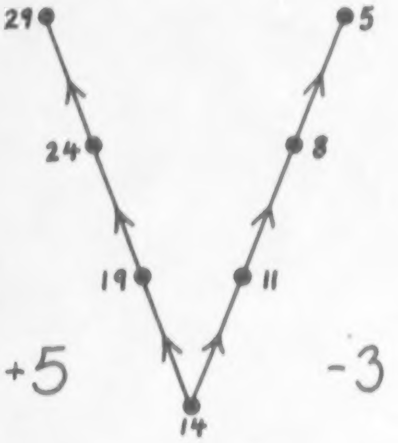


Write any numbers you wish on the Minicomputer, using exactly two positive checkers and two negative checkers.

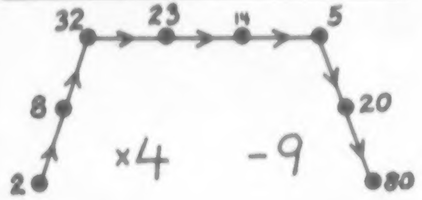


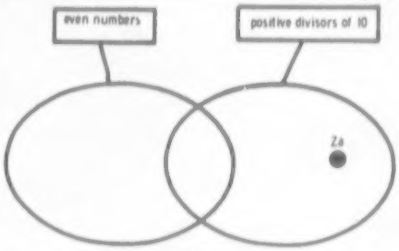
(Many solutions are possible.)

In this picture, 5 is the smallest number and Kurt is the largest number.
Who is Kurt? 29
Label all the dots.

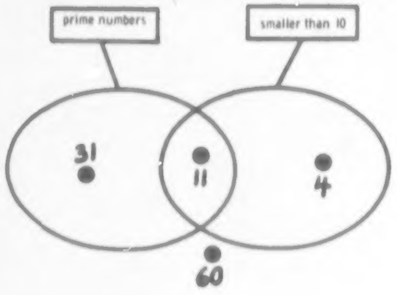


Za is a secret number.
Za is in this arrow picture and in this string picture
Who is Za? 5



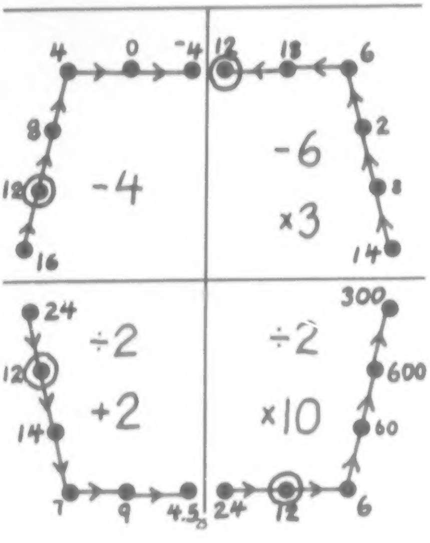


Label the dots. Many answers are possible.

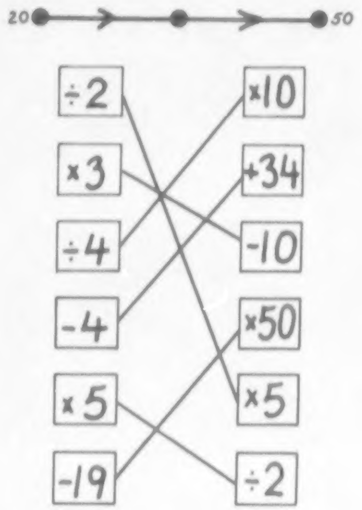


(Many solutions are possible.)

12 is in each of these pictures. In each one circle its dot.
Label all the dots.

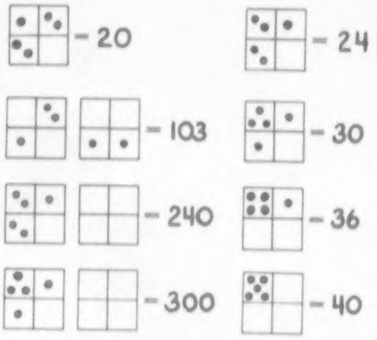


Pair the tags.



26

Put these numbers on the Minicomputer with exactly five checkers.

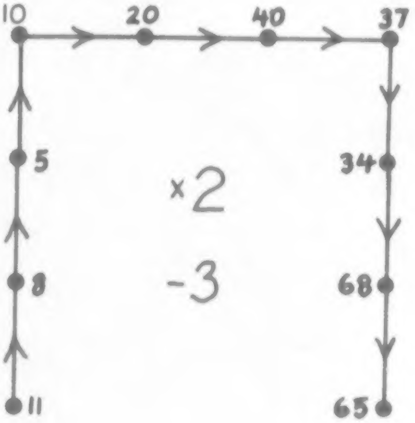


27

(Other solutions are possible.)

First clue

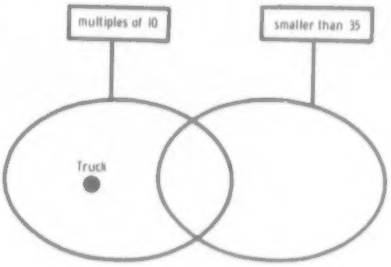
Truck is in this arrow picture.



28

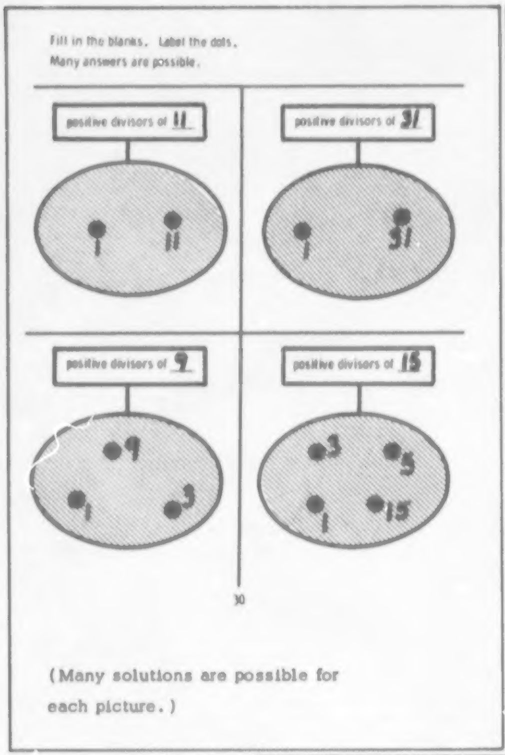
Second clue

Truck is in this string picture.



Truck is 40.

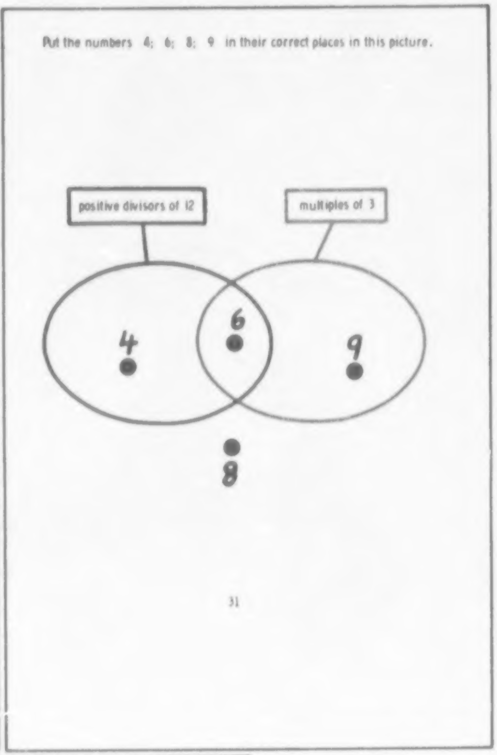
29



Fill in the blanks. Label the dots. Many answers are possible.

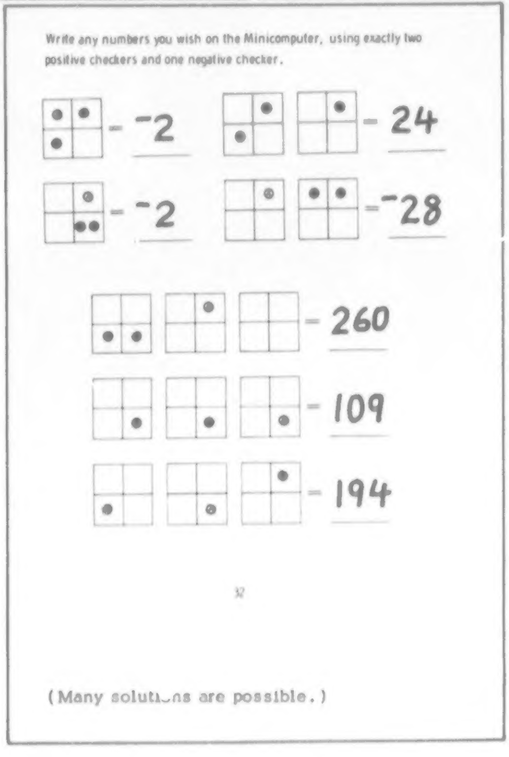
30

(Many solutions are possible for each picture.)



Put the numbers 4; 6; 8; 9 in their correct places in this picture.

31



Write any numbers you wish on the Minicomputer, using exactly two positive checkers and one negative checker.

32

(Many solutions are possible.)

A Set of Problems No. 1 ***

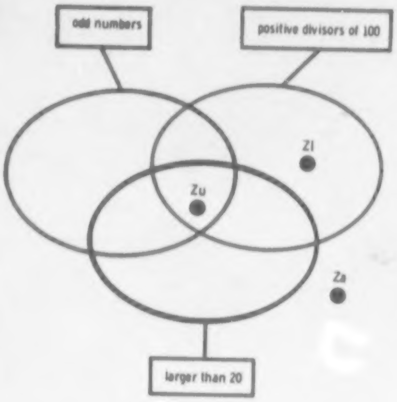
Zi, Za and Zu are three of these numbers:

7; 5; 10; 100; 6; 25; 35; 40.

Who is Zi? 10

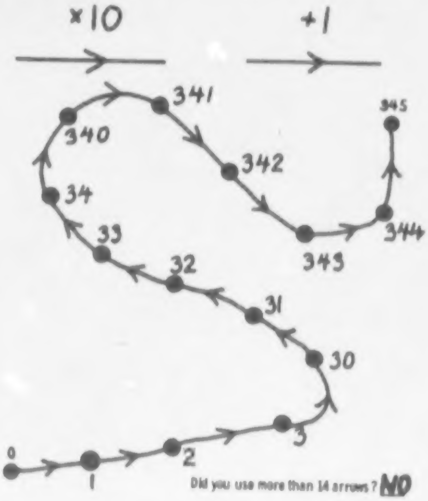
Who is Za? 6

Who is Zu? 25



2

Build a red-green road from 0 to 345 using as few arrows as possible.

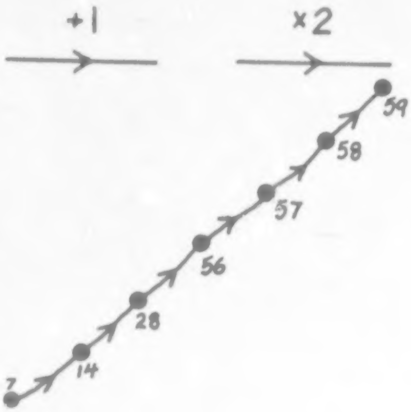


Did you use more than 14 arrows? NO

3

Crack is the smallest possible ending number of a red-green road starting at 7 and using exactly three red arrows and three green arrows.

Who is Crack? 59



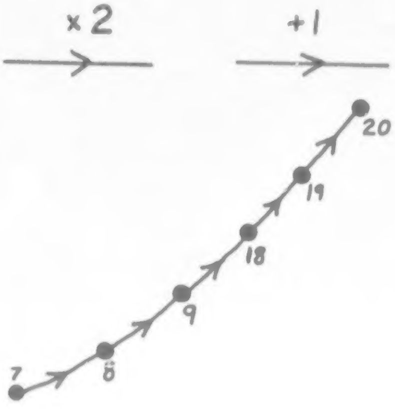
4

Truc is a secret number.

Truc is the ending number of a red-green road starting at 7 and using exactly one red arrow and four green arrows.

Truc can be put on the Minicomputer with exactly one checker.

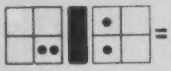
Who is Truc? 20

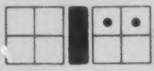


5

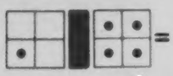
Zig is the smallest and Zag is the largest of these five numbers.

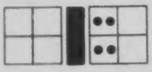
Who is Zig? 1.2 Who is Zag? 3.5


= 3


=1.2

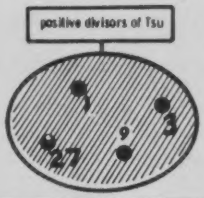
$\boxed{5\times0.25} = 1.25$


= 3.5


= 2

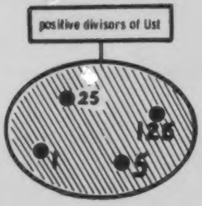
Tsu is a secret number.

Who is Tsu? 27



Ust is a secret number.

Who is Ust? 125



Note: The dots in these two strings can be labeled in different orders, but the numbers involved must always be the same.

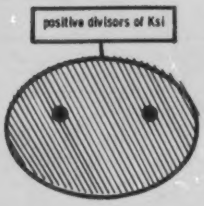
Try to find the eight positive whole numbers smaller than 30 that cannot be put on the Minicomputer using exactly two checkers (positive or negative).

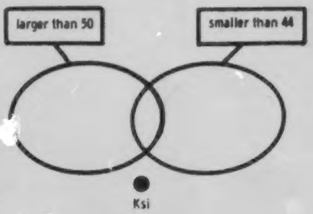
13 ; 15 ; 17 ; 23 ; 25 ; 26 ; 27 ; 29 .

Ksi is a secret number.

These two pictures will help you to find who Ksi is.

Ksi is 59

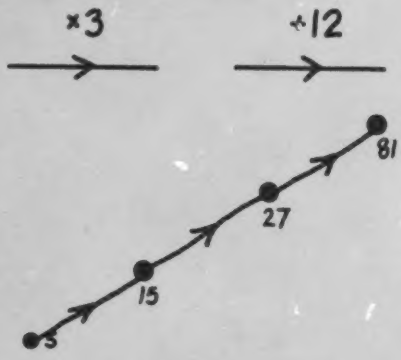




Ku is a secret number.
Ku is the ending number of a red-green road starting at 5 and using exactly one green arrow and two red arrows.

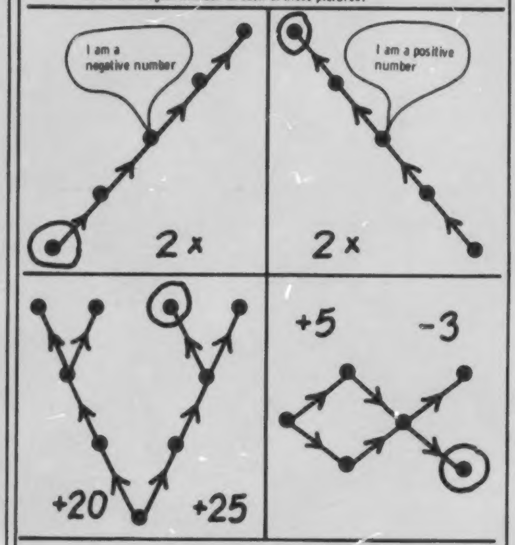
Ku can be shown on the Minicomputer using exactly three checkers (positive or negative).

Who is Ku? 81



10

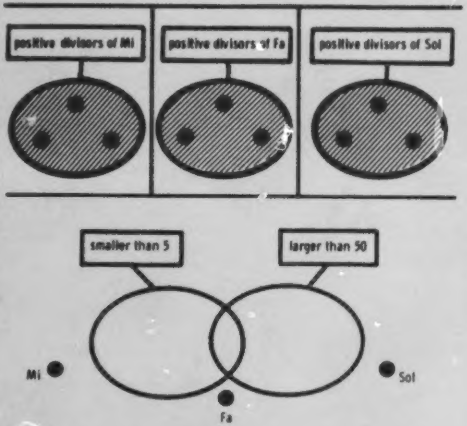
Circle the largest number in each of these pictures.



11

Mi, Fa, and Sol are three secret numbers. These pictures will help you to find them. Mi, Fa, and Sol are

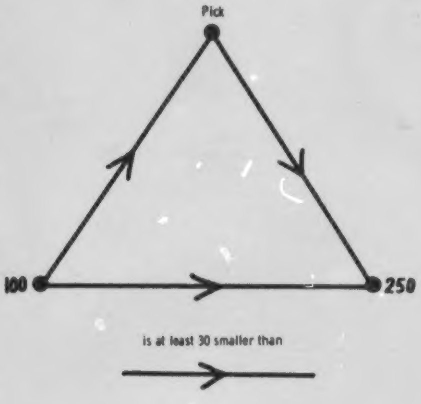
9; 25; 49. *



12

* The order in which Mi, Fa and Sol are written is of no consequence.

Pick is an odd square number.
Who is Pick? 169

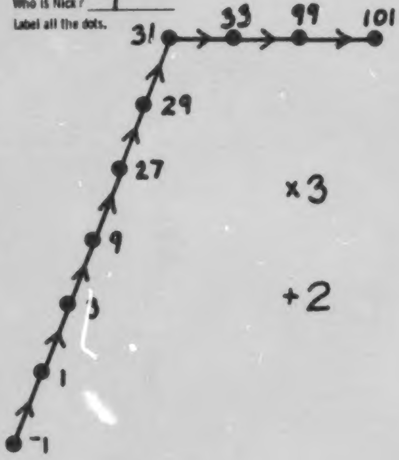


13

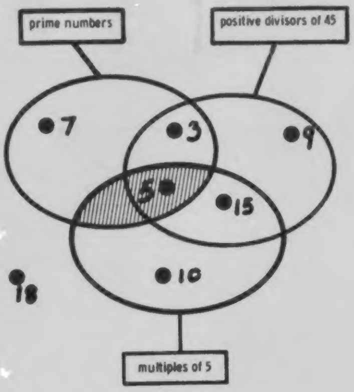
Nick is the only negative integer in this picture. All the other dots are for positive integers.

Who is Nick? ‾1

Label all the dots.



Label the dots. Many answers are possible.



(Many other solutions are possible.)

Try to find the six positive integers smaller than 100 that cannot be put on the Minicomputer using exactly three checkers (positive or negative).

53 ; 55 ; 57 ; 63 ; 65 ; 67 .

ARROW
ROADS

ARROW ROADS

This lesson is included in the workbook strand because it is a combinatorics preparation for some ideas in the "Clinton Street" story-workbook.

T: You have solved many detective stories, and sometimes there was a clue like, "Zip is the ending number of a red-green road starting at some number and using exactly some number of red arrows and some number of green arrows."

Let me give you an example of such a clue.

Write this example on the board and read it to the class.

5x

-2

Zip is the ending number of a red-green road starting at 10 and using exactly two red arrows and exactly two green arrows.

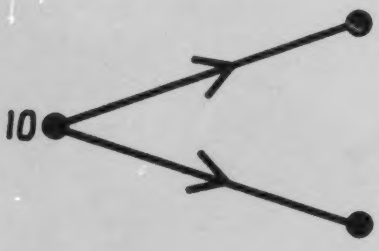
T: Suppose we are solving a detective story and this is one of the clues. Let's see if we can draw a picture that will help us, so we are sure that no arrow road is overlooked.

Draw a dot on the board and label it "10" (starting point).

T: For the moment, don't worry about what the red and green arrows are for. Let's just see if we can draw all the possible arrow roads starting here using two red arrows and two green arrows.

T: If I start an arrow road here (point to "10"), then I can choose either a red arrow or a green arrow for the first arrow of my road.

Draw the following picture on the board.

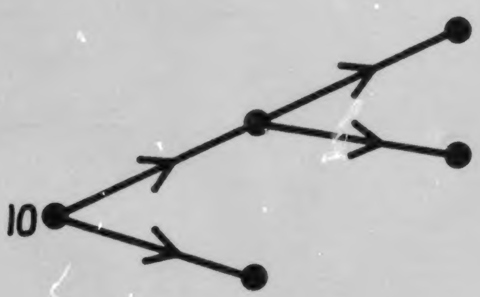


Trace the red arrow as you say:

T: Suppose we choose to take the red arrow. Then what color could the second arrow of the road be?

S: Red or green.

Add this information to your picture.



Trace the second red arrow as you say:

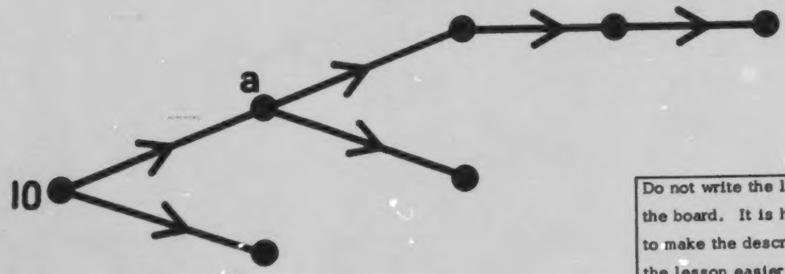
T: Suppose we again choose to take the red arrow.

Trace again the first red arrow followed by the second red arrow and say:

T: We have taken a red arrow for the first arrow of a road and another red arrow for the second arrow. What color could the third arrow be?

S: The third arrow must be green, because we have already used two red arrows. Also, the fourth arrow must be green. We must have two red arrows and two green arrows.

Extend your drawing.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

Starting at 10, trace the arrow road

red followed by red followed by green followed by green.

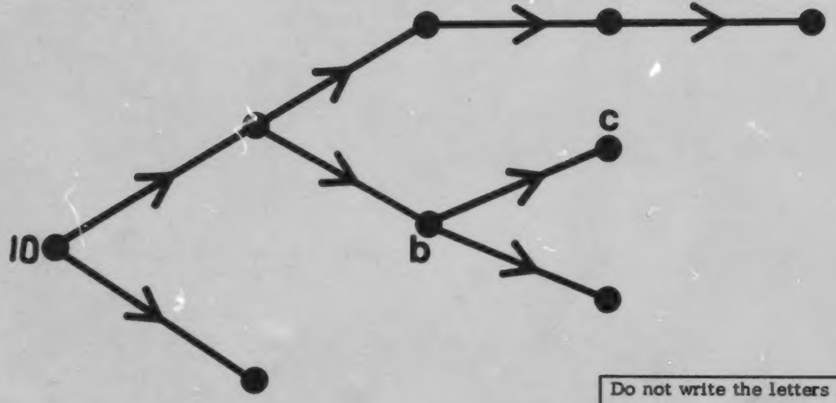
T: We have completed one possible arrow road starting at 10 and using exactly two red arrows and two green arrows.

Start at 10, trace the red arrow from 10 to a, and then trace the green arrow starting at a.

T: If we had taken first the red arrow and then the green arrow as the second arrow, what color could the third arrow be?

S: It could be red or green, because you have only used one of each color thus far.

Add this information.



Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

Trace the third red arrow from b to c.

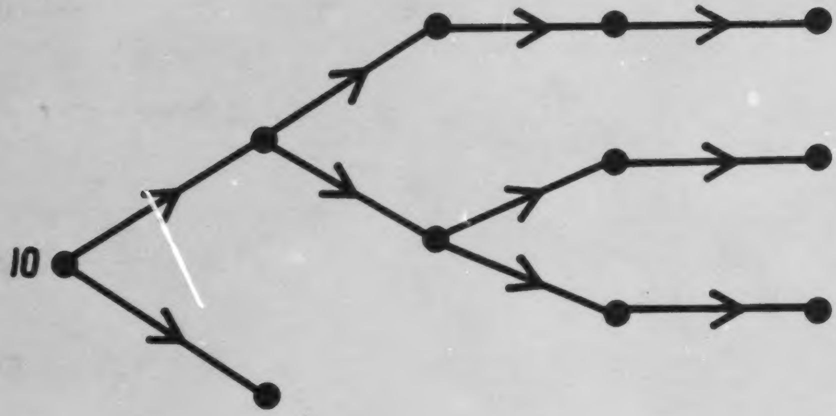
T: Suppose I take red for the third arrow. Then what color could the fourth arrow be?

S: It must be green. You already have two red arrows.

T: Suppose I take green for the third arrow. Then what color could the fourth arrow be?

S: Red; you already have two green arrows.

Add these arrows to your picture.



Start at 10 and trace the arrow road

red followed by green followed by red followed by green.

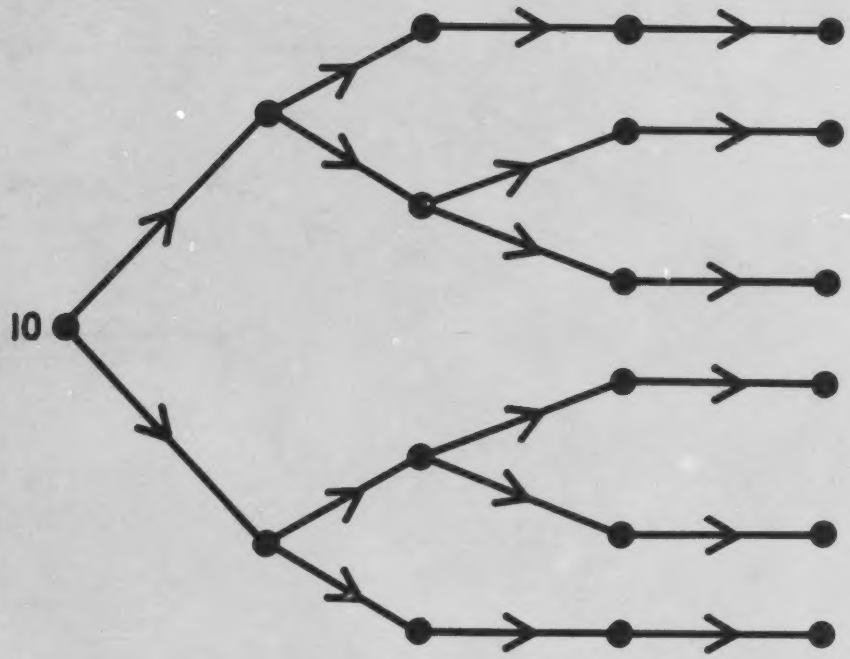
T: This is another arrow road starting at 10 and using exactly two red arrows and two green arrows.

Do the same for the arrow road

red followed by green followed by green followed by red.

4 W

In a similar, but somewhat quicker way, complete the bottom half of the picture.



T: How many red-green roads are there using exactly two red arrows and two green arrows?

S: Six.

T: This picture we have drawn is called a tree diagram of all the possible red-green roads.

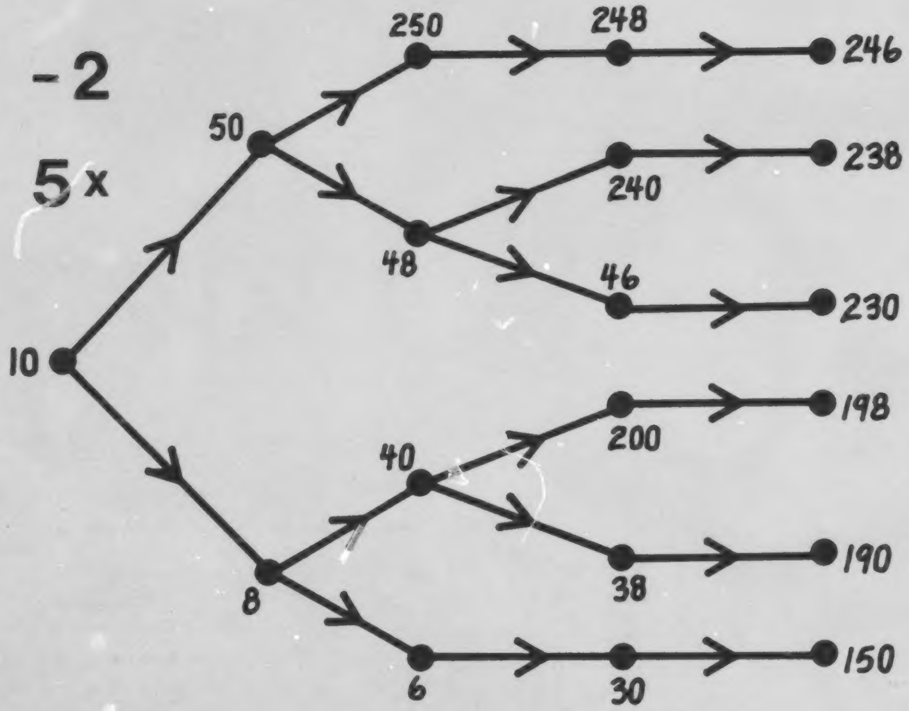
T: How does this tree help us with the clue about Zip?

S: Zip is at the end of one of the roads.

T: Very good. In this clue, the red arrow is for 5 x and the green arrow is for - 2.

Write "5 x" in red and "- 2" in green on the board and have students label the dots.

A completed tree is shown below.



T: What numbers could Zip be?

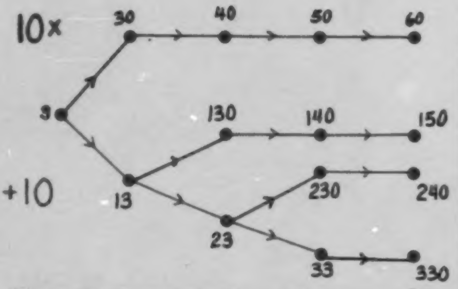
S: 246; 238; 230; 198; 190; and 150.

Worksheets N97, N98, N99, and N100 are available.

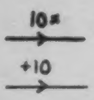
Name ____________________

N 97 *

Label the dots.



How many roads are there using one red arrow and three blue arrows? 4



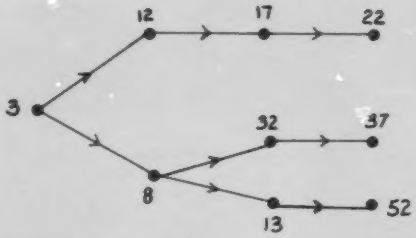
Deck is the ending number of a red-blue road starting at 3 and using one red arrow and three blue arrows.

Deck could be 60, 150, 240, or 330.

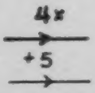
Name ____________________

N 98 **

Draw a picture showing all the red-blue roads using one red arrow and two blue arrows.



How many roads did you find? 3



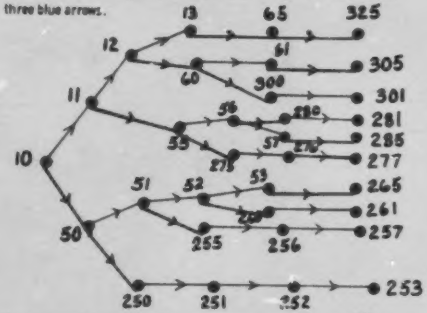
Chex is the ending number of a red-blue road starting at 3 and using one red arrow and two blue arrows.

What numbers could Chex be? 22, 37, 52

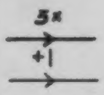
Name ____________________

N 99 ***

Draw a picture showing all the red-blue roads using two red arrows and three blue arrows.



How many roads using two red arrows and three blue arrows? 10



Tax is the ending number of a red-blue road starting at 10 and using two red arrows and three blue arrows.

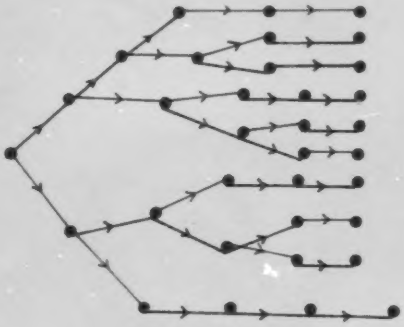
Tax could be: 325; 305; 301; 281; 285; 277; 265; 261; 257; 253

Name ____________________

N 100 ****

How many roads using three red arrows and two blue arrows are there? 10

If you want to check your answer, draw a picture showing all the roads.



How many roads using three red arrows and three blue arrows are there? 20

If you want to check your answer, draw a picture showing all the roads. (Use a separate sheet of paper.)

See Worksheet N 104 for a picture.

A SET OF PROBLEMS NO. 2

A SET OF PROBLEMS NO. 2

This workbook series has three levels of difficulty (* and ** in one booklet and *** in another booklet). It is suggested that all the students begin with level * and continue through as many levels as they can handle during the three lessons scheduled for this series. Not all students are expected to complete all three levels of this workbook series. Only the faster students will reach the most challenging level (***).

Lesson 1

Give each student a copy of the workbook "A Set of Problems - No. 2" levels * and **, and let them work on their own.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

Lesson 2

T: Today we have a detective story about a secret number called "Pop".

First Clue

T: If I put Pop on the display of a calculator and press $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ and so on, 30 will appear on the display. What numbers could Pop be? Try to think of some decimal numbers[*], too.

Give your students a few minutes to explore the numbers that Pop could be.

S: Pop could be 5.

T: Why?

S: If you count by fives, you will get to 30.

T: Very good. So Pop could be 5. What are some other numbers Pop could be?

S: 1, because if I count by ones I can get to 30.

S: 0.25, because I can get to 1 and if I can get to 1, then I can get to 30.

---

[*] What we mean here is actually "noninteger decimal numbers" (e.g., 2.5; 0.15; etc.) but this phrase is much too cumbersome at this level, hence, we say simply "decimal numbers".

Continue in this manner for other numbers that Fop could be. It is helpful with the later clues if you make a list on the board of numbers that Fop could be. In our development classes the students found that Fop could be 30; 15; 10; 7.5; 6; 5; 3; 2.5; 1; 0.5; 0.30; 0.05; and so on.

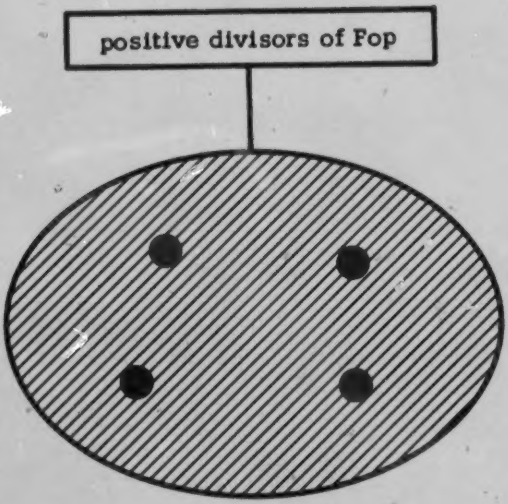
Second clue

T: Fop is an odd whole number.

This clue eliminates the decimals and even numbers from your list of numbers that Fop could be. The class should conclude that Fop could be 15, 5, 3, or 1.

Third clue

Draw this string picture.



T: What does the string picture tell us about Fop?

S: Fop has exactly four positive divisors.

S: Pop cannot be 5 because 5 has two positive divisors.

S: 3 has two positive divisors.

S: 1 has exactly one positive divisor.

S: 15 has four positive divisors: 1; 3; 5; and 15.

The class should conclude that Pop is 15.

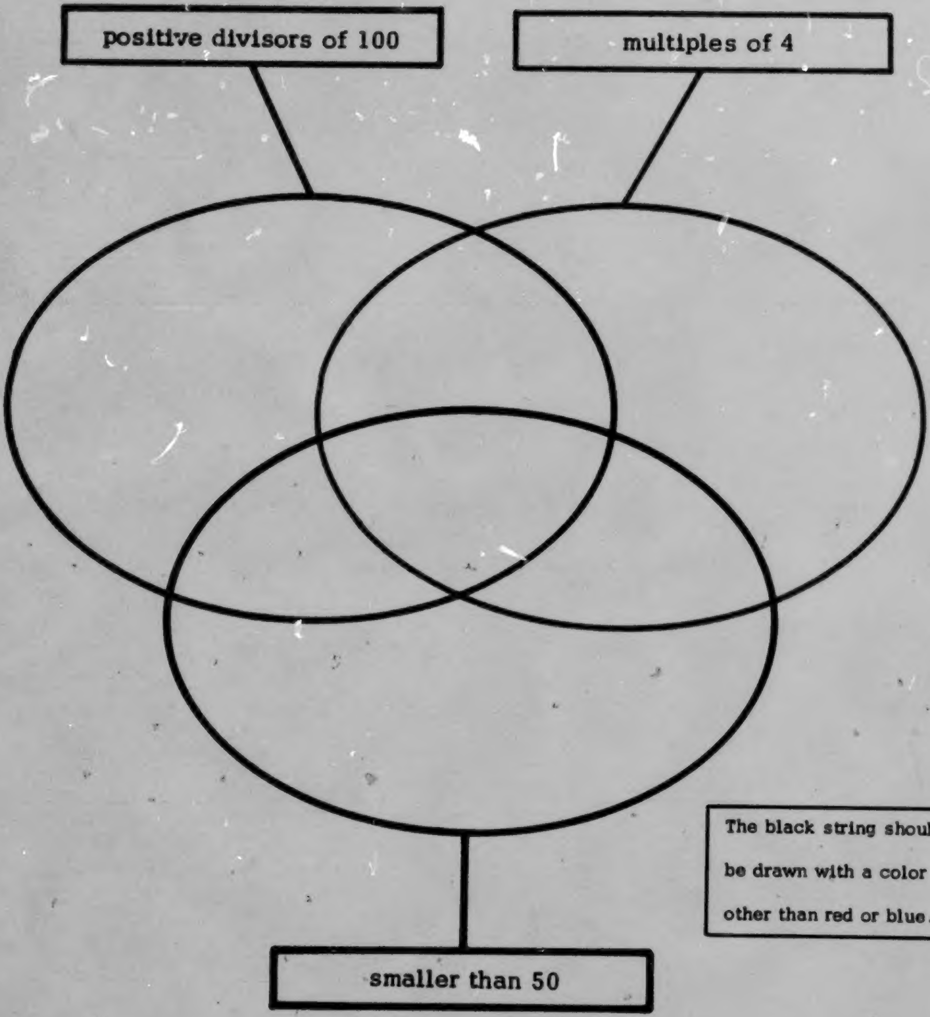
Return the students' copies of "A Set of Problems - No. 2" and ask them to correct their mistakes (if any), then to continue working in their workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem. Any student who finishes a workbook can be given a copy of the next level workbook.

At the end of the lesson, collect the workbooks and have them ready to return for Lesson 3.

Lesson 3

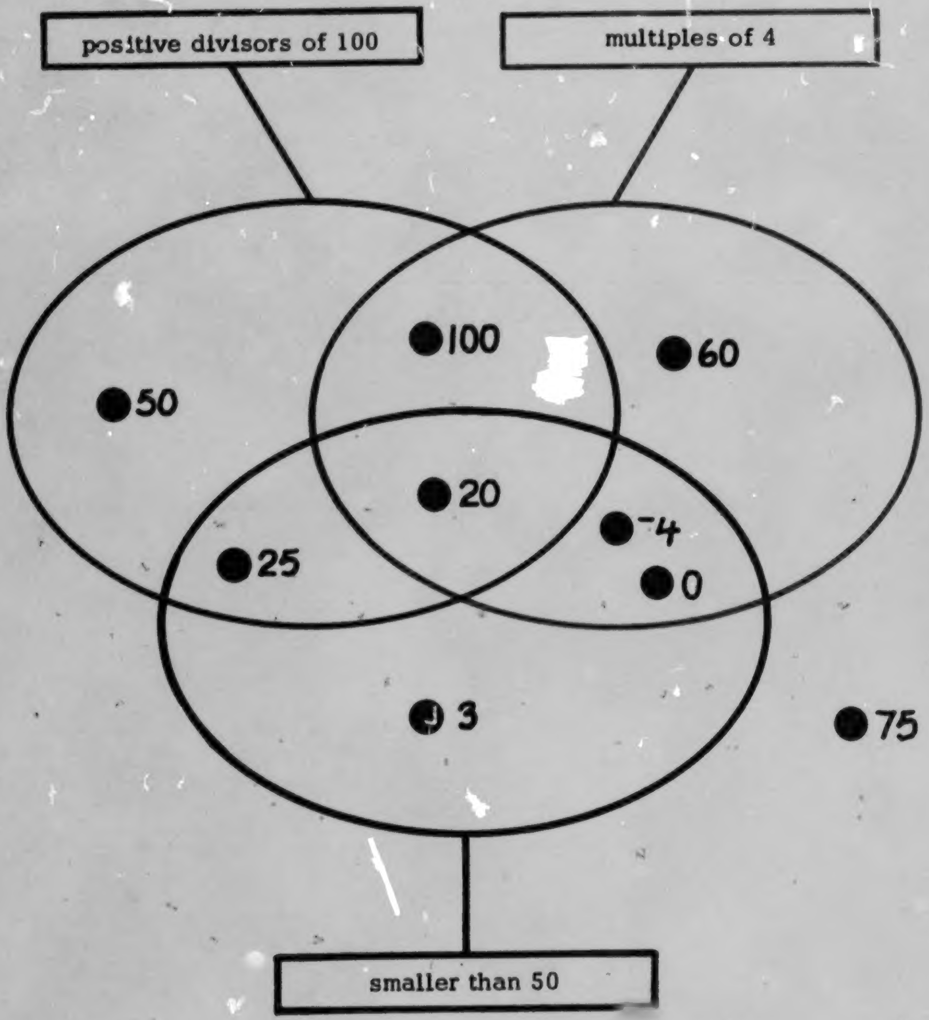
Draw this string picture.



The black string should be drawn with a color other than red or blue.

T: Where would 25 be in this string picture? Is 25 a positive divisor of 100? (Yes) Is 25 a multiple of 4? (No) Is 25 smaller than 50? (Yes) Who can locate 25 in this picture?

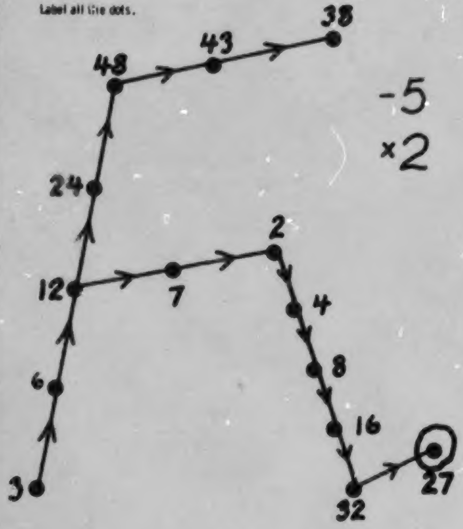
Let a student come to the board and place 25 in the string picture. Repeat the exercise for 20, 50, 100, 75, 0, ⁻4, etc. A completed string picture might look like this.



Return the workbooks and ask the students to correct their mistakes (if any), then to continue working.

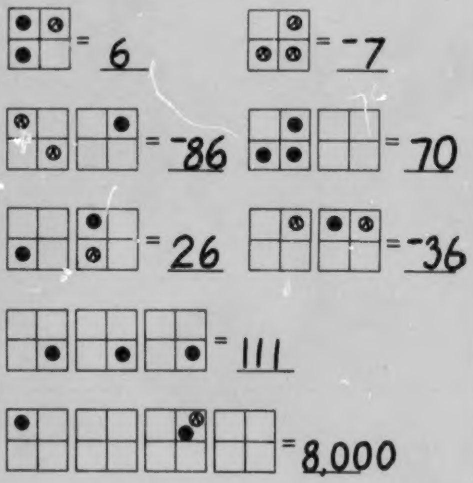
A Set of Problems No. 2 * and **

27 is in this picture. Circle its dot.
Label all the dots.



2

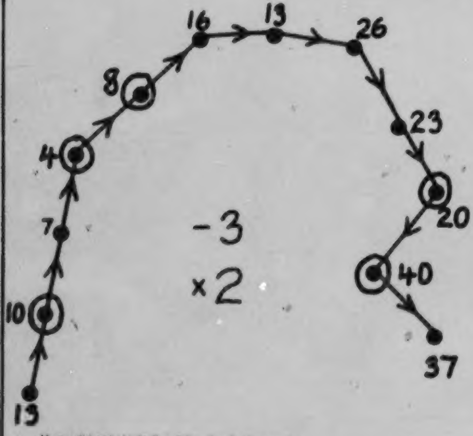
Write any numbers you wish on the Minicomputer using exactly three checkers (positive or negative).



3

(Many solutions are possible.)

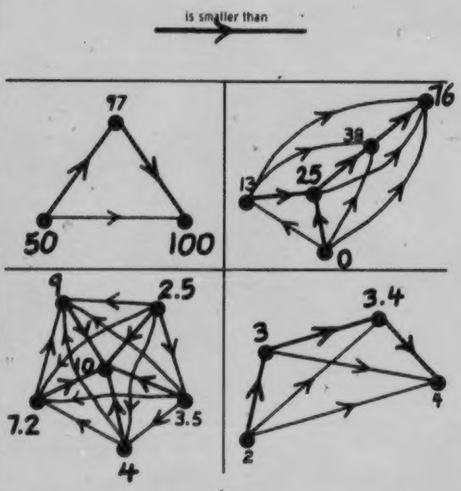
Label all the dots.
Circle each dot for a number that can be put on the Minicomputer with exactly one checker.



If you did not circle five dots, check your work.

4

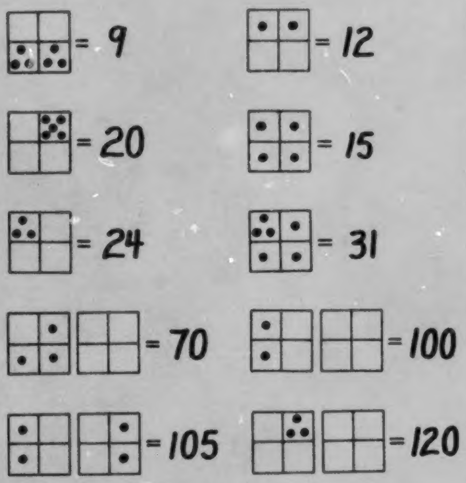
Label the dots. Many answers are possible.
Draw the missing red arrows.



5

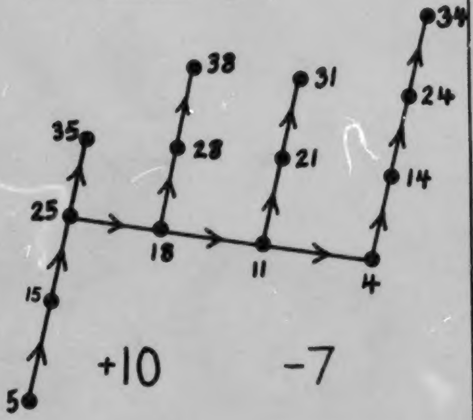
(Many solutions are possible.)

Write these numbers on the Minicomputer.



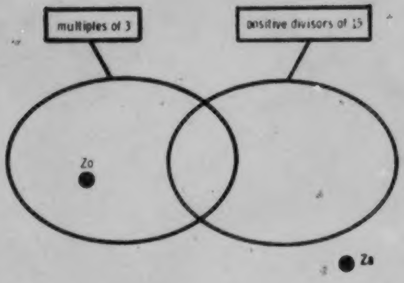
(Several solutions are possible.)

Nick is the largest number and Nack is the smallest number in this arrow picture.
Label all the dots.
Who is Nick? 38
Who is Nack? 4



Za and Zo are two of these numbers:

1; 3; 5; 6; 7; 15.

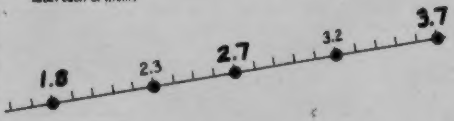


Za is 7.
Zo is 6.

Zan is a secret number.

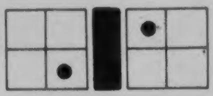
First clue

One of these five dots is for Zan.
Label each of them.



Second clue

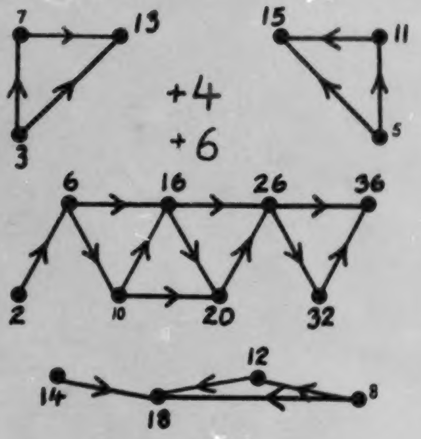
Zan can be put on the Minicomputer with exactly one positive checker on each board.



Zan is 1.8.

Label all the dots.

Use blue to draw as many + 10 arrows as you can.



If you did not draw seven + 10 arrows, check your work.

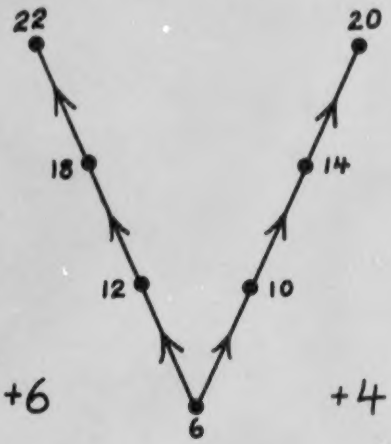
10

(All of the blue arrows are drawn in black.)

20 and 22 are the two largest numbers in this picture and Tog is the smallest one.

Label all the dots.

Who is Tog? 6

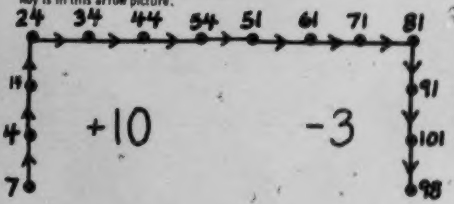


11

Roy is a secret number.

First clue

Roy is in this arrow picture.



Second clue

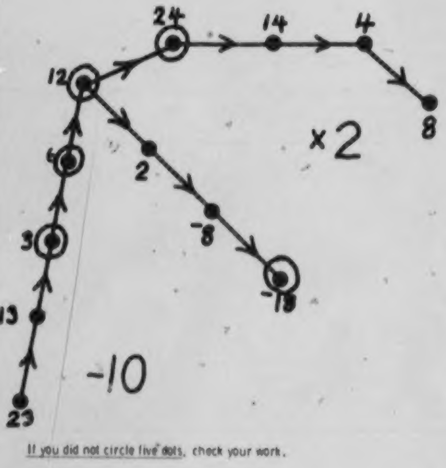
The double of Roy is larger than 200.

Roy is 101.

12

Label all the dots.

Circle each dot for a multiple of 3.



If you did not circle five dots, check your work.

13

Find the four numbers that can be put on the ones' board of the Minicomputer with three positive checkers, each on a different square.

14 ; 7 ; 13 ; 11 .

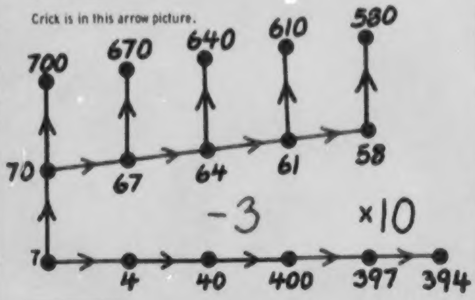
If you add up all these numbers and then multiply the result by 10, you will get Mick.

Mick is 450 .

Crick is a secret number.

First clue

Crick is in this arrow picture.

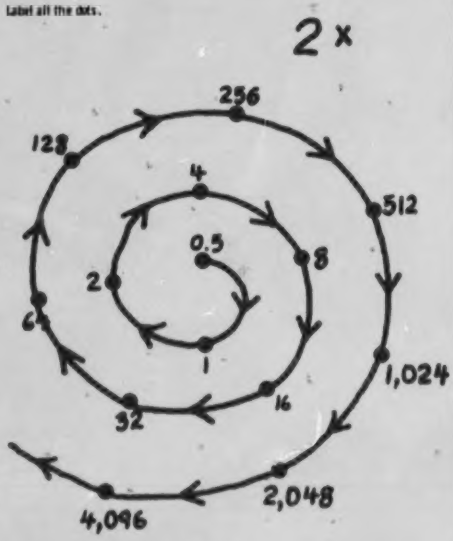


Second clue

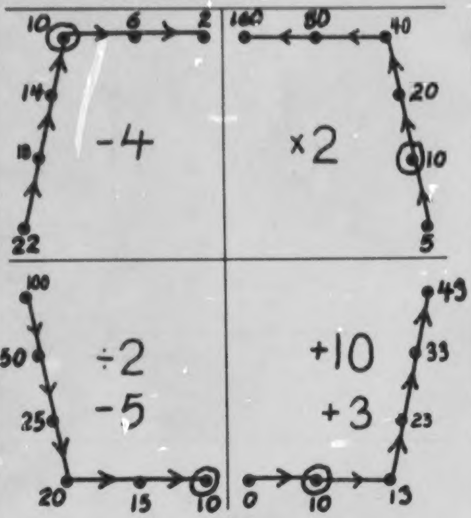
Crick is larger than 50 and can be put on the Minicomputer with exactly one checker.

Crick is 400.

Label all the dots.

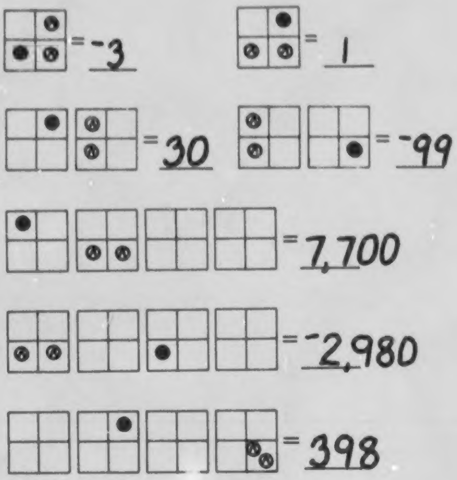


Label all the dots.
10 is in each of these pictures. In each one circle its dot.



18

Write any numbers you wish on the Minicomputer using exactly one positive checker and two negative checkers.



= -3

= 1

= 30

= -99

= 7,700

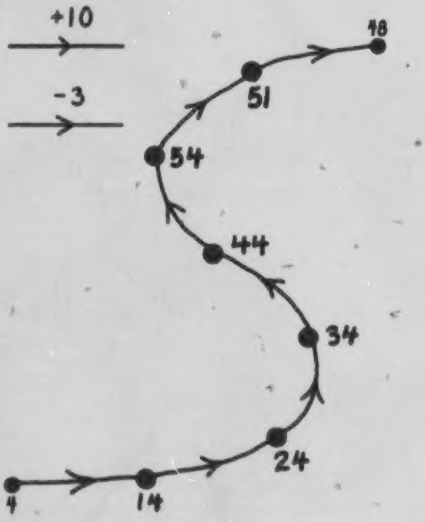
= -2,980

= 398

19

(Many solutions are possible.)

Build a red-green road from 4 to 48.

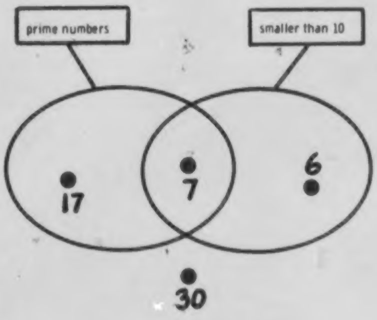


20

(Several other solutions are possible.)

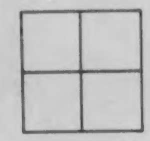
Put these numbers in this string picture.

6; 30; 7; 17.



21

Find the ten numbers that can be put on the ones' board of the Minicomputer using exactly two positive checkers.



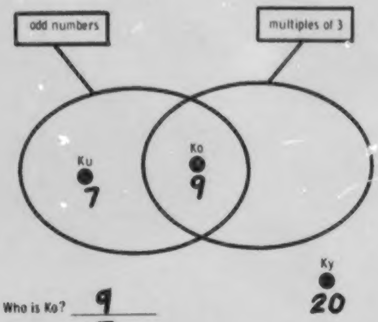
2; 3; 4; 5; 6; 8; 9; 10; 12; 16.

If you add up all these numbers and then double the result, you will get Zo.

Zo is 150.

Ko, Ky and Ku are three of these numbers:

12; 9; 18; 7; 20



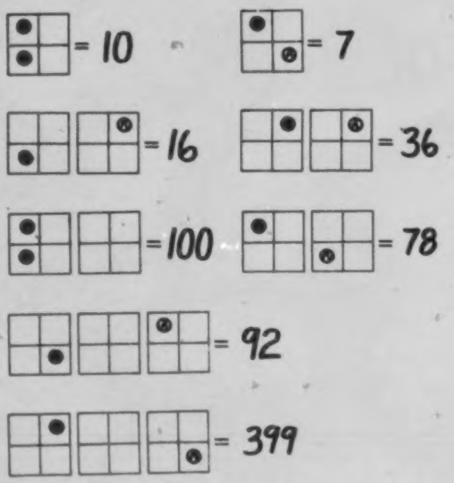
Who is Ko? 9

Who is Ku? 7

Who is Ky? 20

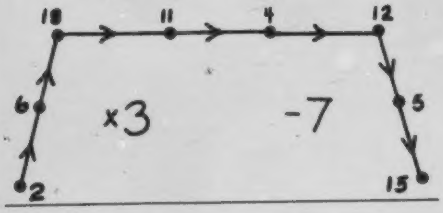
Write their names on the picture.

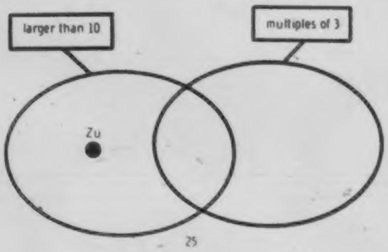
Write each of these numbers on the Minicomputer using exactly two checkers (positive or negative).



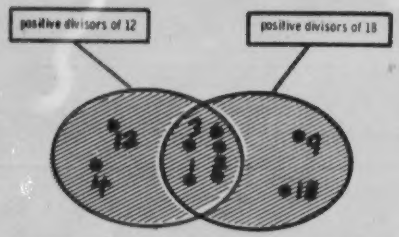
(Other solutions are possible.)

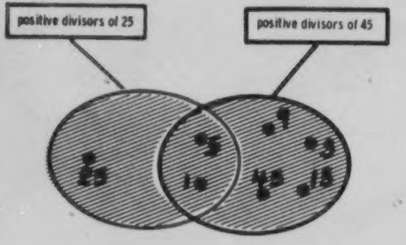
Zu is a secret number.
Zu is in this arrow picture and in this string picture.
Label all the dots.
Who is Zu? 11





Label the dots.

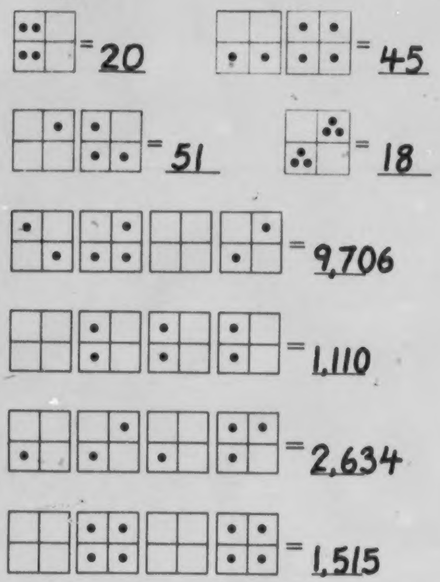




26

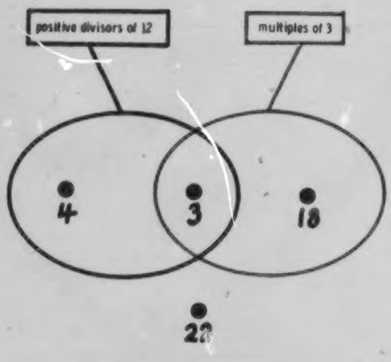
(The labels may be arranged in various ways within the same region.)

Complete:



27

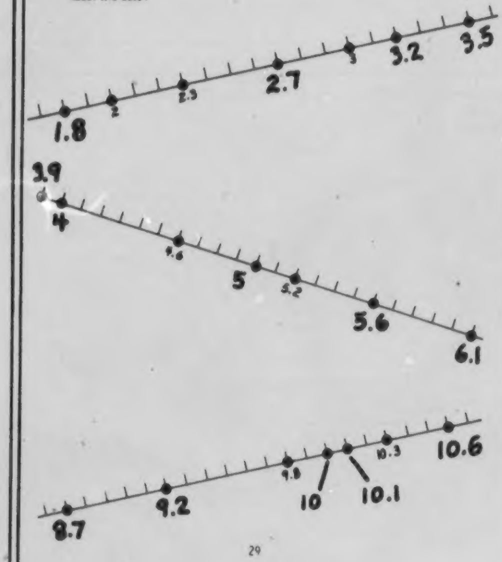
Label the dots. Many answers are possible.



28

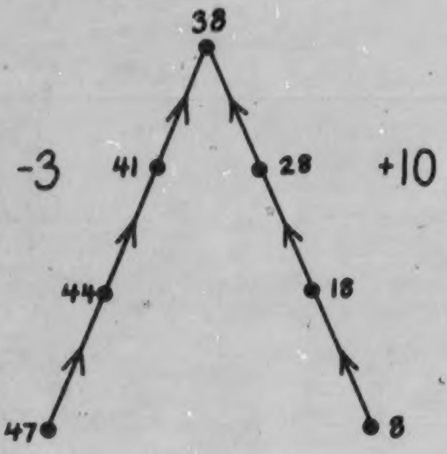
(Many solutions are possible.)

Label the dots.

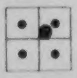


29

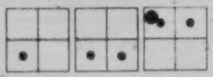
In this picture, 8 is the smallest number and Kim is the largest number.
Label all the dots.
Who is Kim? 47



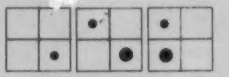
Add one checker to this Minicomputer board to get 19.



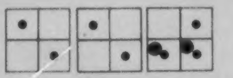
Add one checker to these Minicomputer boards to get 250.



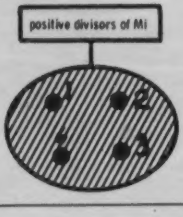
Add two checkers to these Minicomputer boards to get 200.



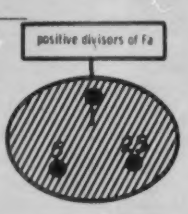
Add two checkers to these Minicomputer boards to get 1,000.



Mi is a secret number.
Label all the dots.
Who is Mi? 6



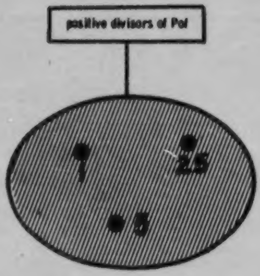
Fa is a secret number.
Label all the dots.
Who is Fa? 25



(The labels may be arranged in various ways.)

A Set of Problems No. 2 ***

Pol is a secret odd number.
If you put Pol on the display of your hand-calculator and press
$\boxed{+}$ $\boxed{=}$ $\boxed{=}$ . . . , 100 will appear on the display.

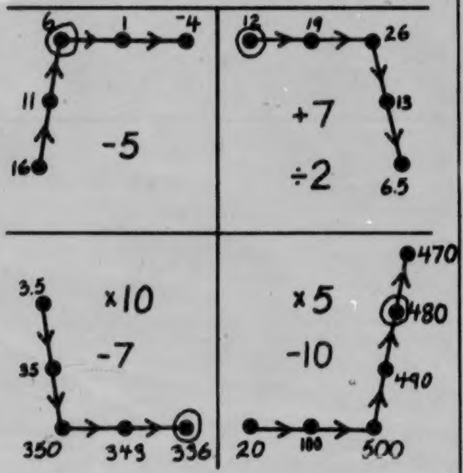


Pol is 25.

2

(The dots can be labeled in different orders but always with the same numbers.)

Label all the dots.
There is exactly one multiple of 3 in each of these pictures. In each one circle its dot.



3

Write the eight prime numbers that are smaller than 20.

2 ; 3 ; 5 ; 7 ; 11 ; 13 ; 17 ; 19 .

If you add up all these numbers and then divide the result by 7, you will get Za.

Za is 11.

Write the eleven prime numbers that are smaller than 35.

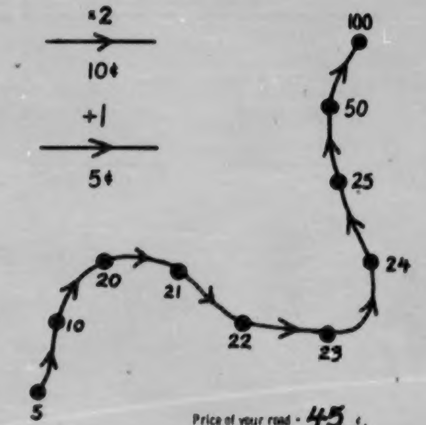
2 ; 3 ; 5 ; 7 ; 11 ; 13 ; 17 ; 19 ; 23 ; 29 ; 31 .

If you add up all these numbers and then divide the result by 10, you will get Zen.

Zen is 16.

4

Build a red-green road from 5 to 100.
For each red arrow you have to pay 10¢.
For each green arrow you have to pay 5¢.
Try not to spend more than 50¢.

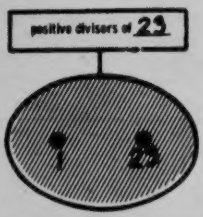


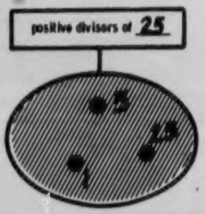
Price of your road = 45 ¢.

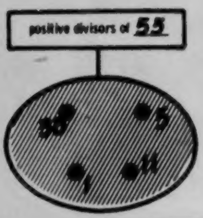
5

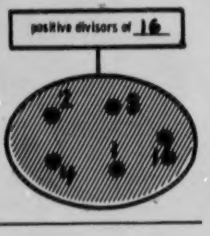
(Other solutions are possible.)

Fill in the blanks and label the dots.
Many answers are possible.





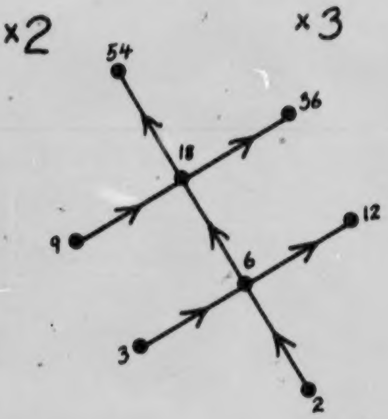


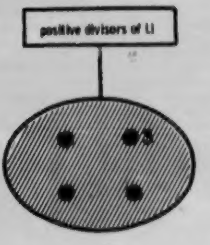


(Many other solutions are possible for each picture.)

Put all of these numbers in the arrow picture.

2; 3; 6; 9; 12; 18; 36; 54



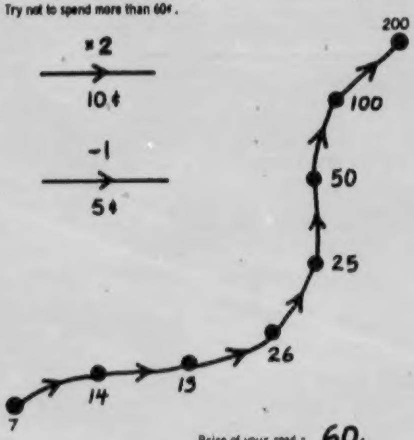


Write at least four numbers that Li could be.

6 ; 15 ; 21 ; 27

(Many possible solutions.)

Build a red-green road from 7 to 200.
For each red arrow you have to pay 10¢.
For each green arrow you have to pay 5¢.
Try not to spend more than 60¢.



Price of your road = 60¢.

If you put Zig on the display of your hand-calculator and press
⊡ ⊡ ⊡ ..., 20 will appear on the display.

Write at least twelve numbers (whole numbers or decimal numbers) that Zig could be.

1 ; 2 ; 4 ; 5 ;

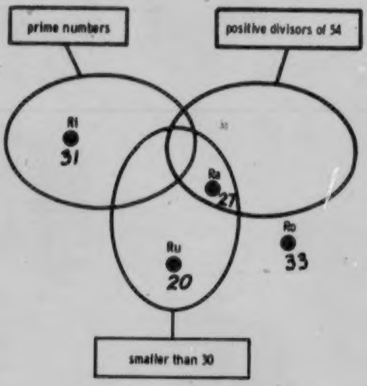
10 ; 20 ; 2.5 ; 0.5 ;

0.2 ; 0.02 ; 0.25 ; 0.04 .

(Many other solutions are possible.)

Ri, Ra, Ro and Ru are four of these numbers:

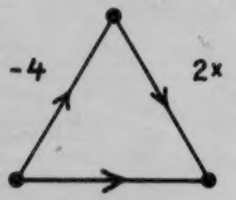
2; 3; 7; 11; 20; 27; 31; 33; 54



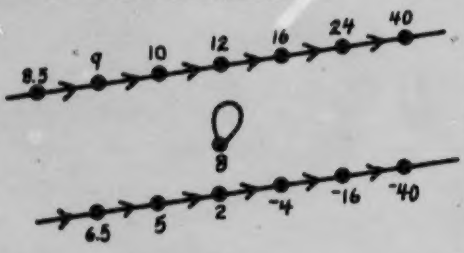
Ri is 31 ; Ra is 27 ; Ro is 33 ; Ru is 20 .

Write their names on the picture.

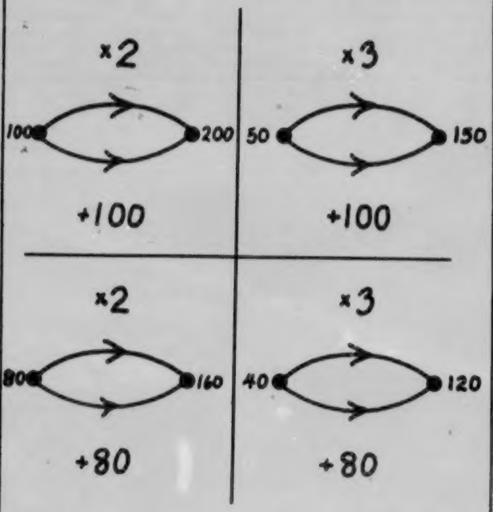
Here is the code for the red arrows.

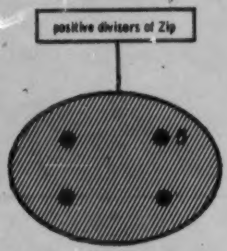


Using this code label the dots in the picture below.



Label the dots.



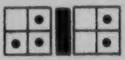


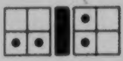
Write at least six numbers that Zip could be.

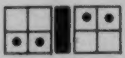
10 ; 15 ; 35 ;

55 ; 65 ; 125 .

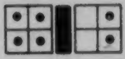
(Many other solutions are possible.)

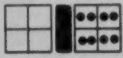

= 7.5


= 4

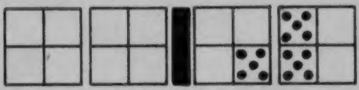

= 4.2 ←

Be careful!
The answer is not 3.12.


= 14.5


= 3

Write 1 on the Minicomputer using exactly 15 positive checkers:



(Other solutions are possible for the last problem.)

If I put Clo on the display of my hand-calculator and press $\boxed{\times}$ $\boxed{2}$ $\boxed{+}$ $\boxed{4}$ $\boxed{=}$, I get a number between 8 and 9.
Write at least twelve numbers that Clo could be.

2.01 ; 2.1 ; 2.15 ; 2.2 ;

2.25 ; 2.3 ; 2.31 ; 2.35 ;

2.4 ; 2.45 ; 2.49 ; 2.495 .

(Any number between 2 and 2.5 is a solution.)

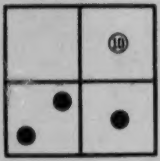
CLINTON STREET

CLINTON STREET

Lesson 1

Distribute a copy of the story-workbook "Clinton Street - Book A" to each child. Ask the students to read pages 4 - 7 and then to try to solve the problem on page 7.

While the students are reading, display one Minicomputer board with this configuration.



Remind the students that we use regular checkers for pennies.

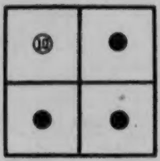
T: What number is on the Minicomputer?

S: 45.

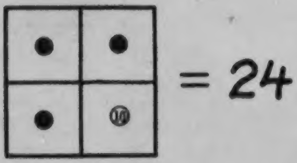
T: Why?

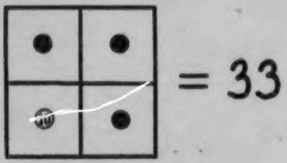
S: 4 × 10 = 40; 2 + 2 = 4 and 1 more is 5; and 40 + 5 = 45.

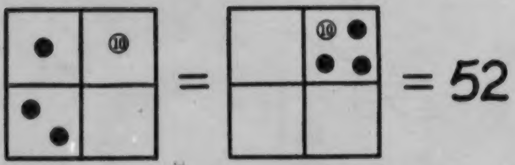
Have the class decode the number on page 9 and check their own work with page 10. Then, collectively decode the following numbers.

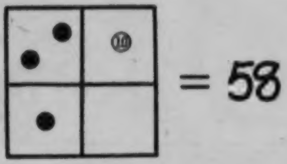


= 87



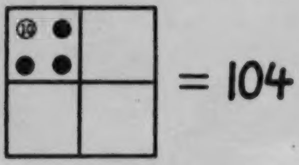




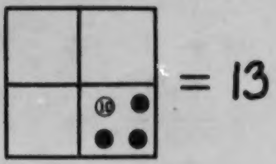


T: I would like someone to put the largest number possible on the Minicomputer using exactly one dime and three pennies.

Let a student put the checkers on the board. The largest number is 104.



Do a similar exercise for the smallest number, 13.



T: Do you think we can represent all the numbers between 13 and 104 on the Minicomputer using exactly one dime and three pennies?

Let the class react to this situation, perhaps trying more examples. If one of the many impossible numbers is not suggested after a few minutes, ask:

T: Who can put 55 on the Minicomputer using one dime and three pennies?

Let several students try to put the checkers on the board. The class should conclude that it is impossible to put 55 on the Minicomputer using one dime and three pennies.

Collect the story-workbooks. Worksheets N101, N102, and N103 are available for the rest of the lesson.

As the faster students finish the worksheets, challenge them with this problem (see page 11 in the story-workbook).

***

Try to make a list of all the numbers that can be put on the ones' board with one dime and three pennies.

This is a difficult problem, but you will probably have a few students accept the challenge. A hint would make the problem easier and would help sustain interest. For instance, suggest that the students first consider what numbers can be put on the Minicomputer using three pennies (there are 17 numbers), and then using three pennies and one dime. Those that construct this list might reason that there are 68 such numbers, since there are 68 such Minicomputer configurations. But they should find that there are many duplicates.

8W

There are 60 numbers listed below that can be put on the Minicomputer with one dime and three pennies.

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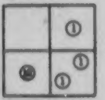
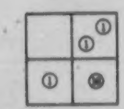
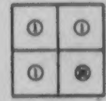
104

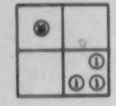
Name ____________________ **N 101 ***

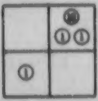
Complete:

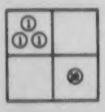
 = 26

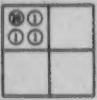
 = 20

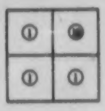
 = 24

 = 83

 = 50

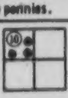
 = 34

 = 104

 = 51

Name ____________________ **N 102 ***

Write any number you wish on the Minicomputer using exactly one dime and three pennies.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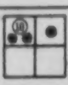
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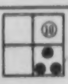
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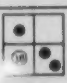
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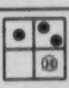
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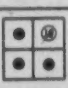
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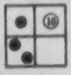
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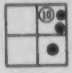
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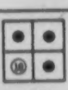
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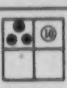
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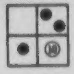
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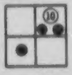
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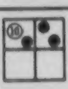
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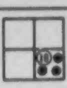
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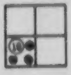
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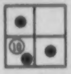
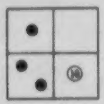
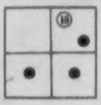

31

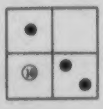
The black dots represent ①-checkers.
Other solutions are possible.

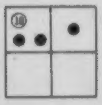
Name ____________________ **N 103 ****

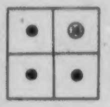
Write these numbers on the Minicomputer using exactly one dime and three pennies.

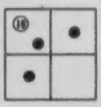
 = 22

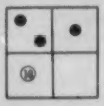
 = 47

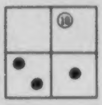
 = 30

 = 100

 = 51

 = 94

 = 40

 = 45

The black dots represent ①-checkers.
Other solutions are possible.

Lesson 2

T: Do you remember the story of Spike that we read some days ago? Who would like to tell it?

Let some students tell the story or part of it. Then, distribute copies of "Clinton Street - Book A" and have the students read and solve the problems from page 12 through page 16.

Read and discuss page 17 with the entire class. Then ask the students to work on pages 18 and 19. When most of the class have completed those pages, continue the collective discussion.

T: Let's make a list of the houses Spike will have to search.

It is likely that some students will not complete page 18. By making the list, the students can add to or check their work. In any event, the class should conclude that Spike will have to search the following seventeen houses: 83; 84; 85; 86; 87; 88; 89; 90; 91; 92; 93; 94; 96; 97; 98; 100; and 104.

T: Three of these numbers can be put on the Minicomputer in two ways. Which ones are they?

S: 86, 90, and 92.

T: Good. Five numbers between 83 and 104 cannot be put on the Minicomputer in this way. Which ones are they?

S: 95, 99, 101, 102, and 103.

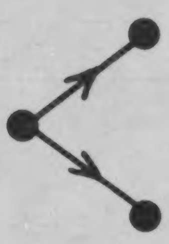
Read pages 20 and 21 together with the class.

T: Spike was not as fortunate as you. He did not know how to draw a tree diagram for this situation. Let's help Spike and draw a tree diagram.

Draw a starting point in the center of the board.

T: If I start here, then I can choose either a red arrow or a blue arrow for the first arrow of my road.

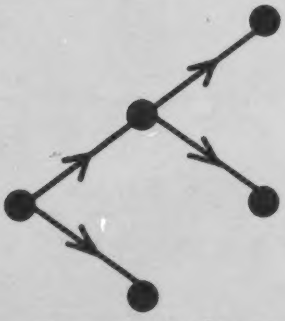
Draw these two arrows on the board.



T: Suppose we choose to start with a red arrow. What color could the second arrow of the road be?

S: Red or blue.

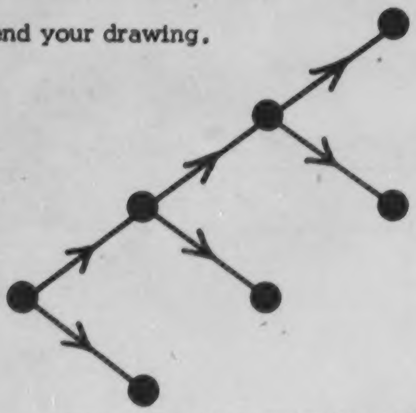
Add this information to your tree.



T: Suppose we choose a red arrow for our second arrow. What color could the third arrow of our road be?

S: Red or blue.

Extend your drawing.

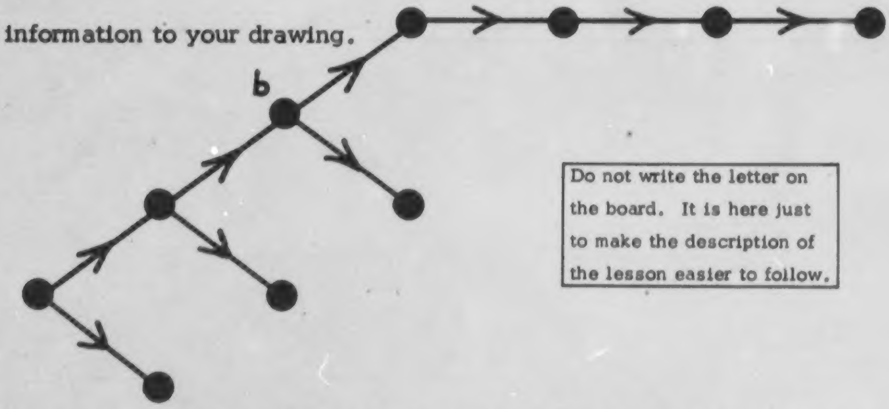


T: Suppose we choose red for our third arrow. What color could the fourth arrow of our road be?

S: It must be blue because you can have only three red arrows.

S: You must draw two more blue arrows to finish the road.

Add this information to your drawing.

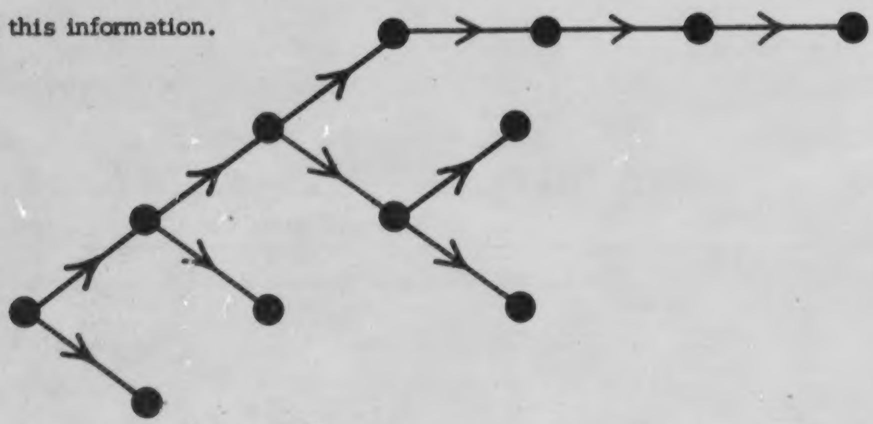


Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

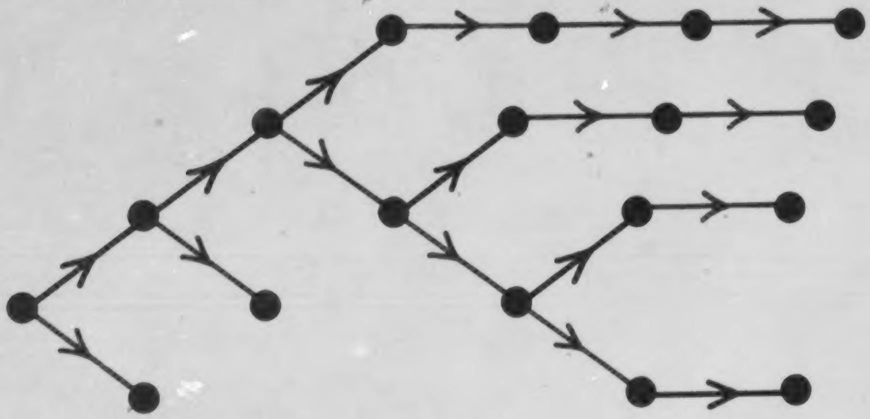
T: Here is one possible arrow road. (Trace the red-red-red-blue-blue-blue arrow road.) It has three red arrows followed by three blue arrows. What if we had chosen a blue arrow for our third arrow? (Point to b and trace the blue arrow starting there.)

S: The next arrow could be red or blue.

Add this information.



Continue in this way completing one or two more roads. A partially completed tree diagram is shown below.



Distribute worksheet N104.

T: Here is the complete tree diagram. Compare this diagram to Spike's work on pages 22 and 23. Which do you prefer?

Accept any student comments. There will probably be a variety of responses.

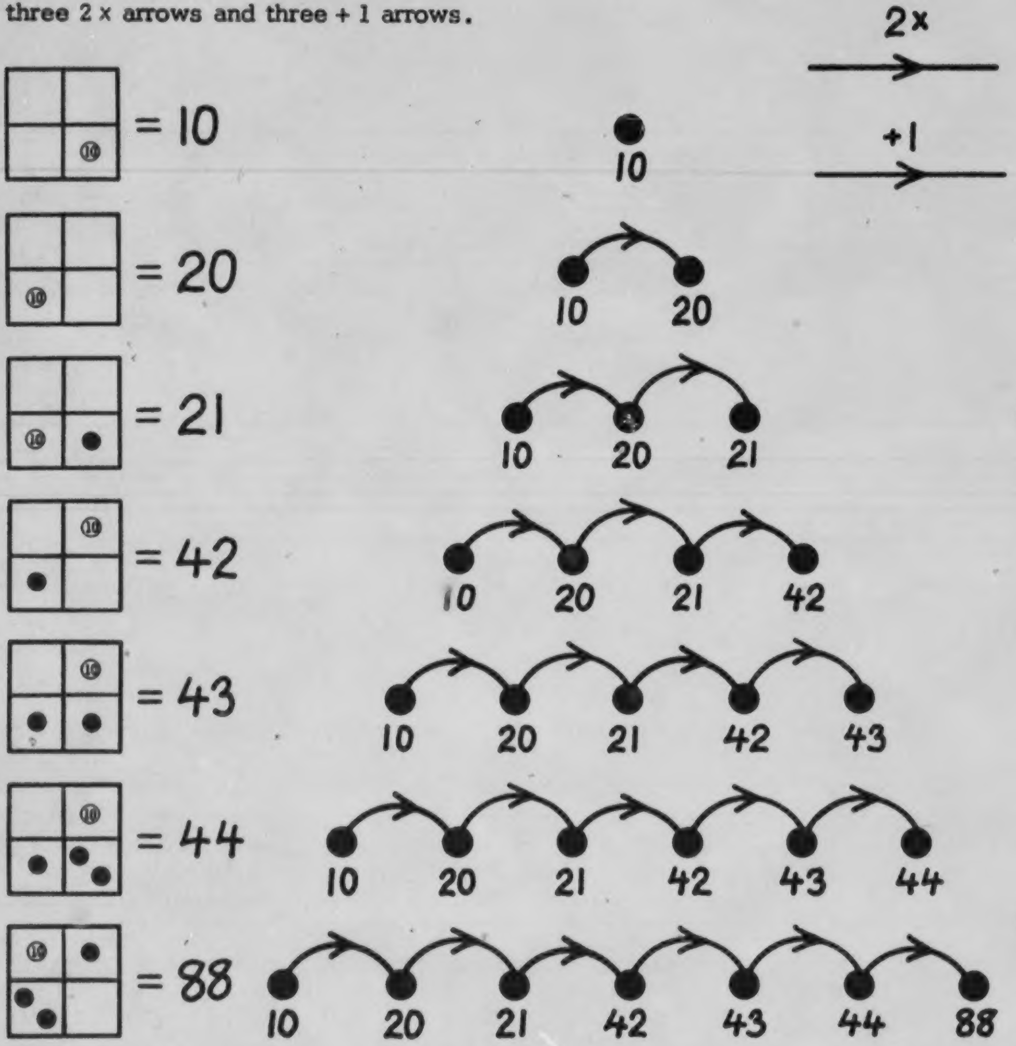
Let the class continue by reading page 24 and labeling the dots either on worksheet N104 or on pages 22 and 23. Students who finish should read and answer the questions on page 25.

Don't forget to collect the books at the end of the lesson.

9W

NOTE: On page 25 of "Clinton Street - Book A", we stated that there was a connection between the roads clue and the Minicomputer clue. One way to think of that connection is suggested here.

Compare the placing of a dime on the 8-square and three pennies anywhere on the Minicomputer to building an arrow road starting at 10 and using exactly three 2 x arrows and three + 1 arrows.

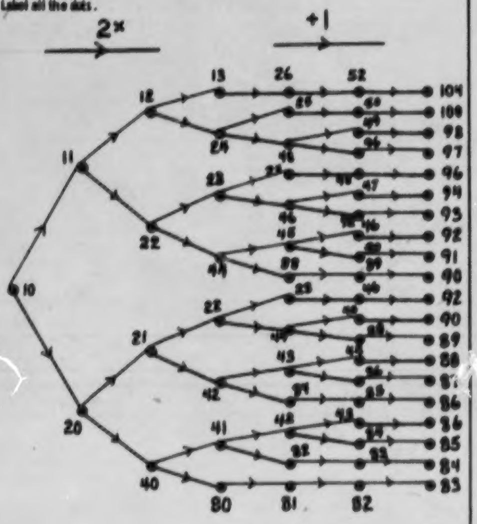


For each number that can be put on the Minicomputer there exists an arrow road and vice versa.

Name

N 104 *

Label all the dots.



Lesson 3

Do not distribute copies of "Clinton Street - Book A" until later. Have a student tell the part of the story read thus far. If necessary, summarize the two clues.

T: Do you remember the story of Clinton Street and the first clue that Spike received?

S: A dime was on the 8-square and three pennies were somewhere on the Minicomputer.

T: Good. Do you remember the smallest possible number that can be put on the ones' board of the Minicomputer in this way?

S: 83.

T: The largest number?

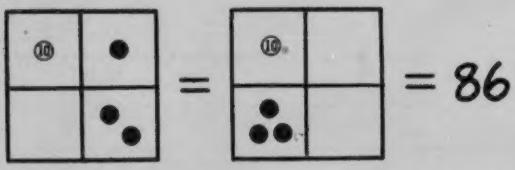
S: 104.

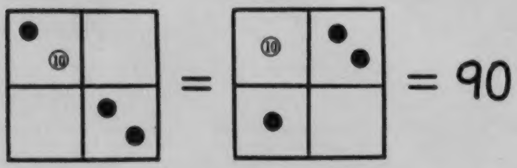
T: There were many numbers in between. Do you remember one number that cannot be put on the Minicomputer in this way? (95, 99, 101, 102, 103)

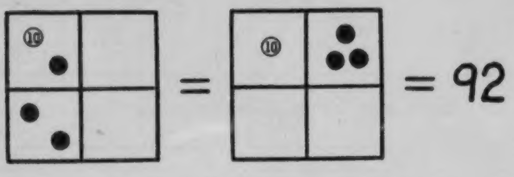
Should a student suggest a number other than one of the five listed above, let someone come up and show the suggested number on the Minicomputer.

T: There were some numbers that could be put on the Minicomputer in two ways using one dime and three pennies. I would like someone to show us those numbers.

Let volunteers come to the board and show the three numbers.



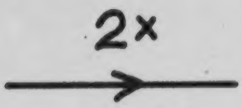


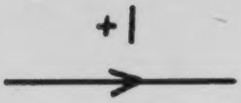


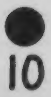
T: Do you remember the second clue about arrow roads?

S: 10 was the starting number of each road. Each road had exactly three red arrows and three blue arrows.

Write this information on the board.







T: I would like each of you to draw at least one arrow road starting at 10 and using exactly three arrows of each color. When you finish, try to solve the two-star problem that I will write on the board.

> **
>
> Draw the two roads that start at 10 and end at 90 using exactly three arrows of each color.

If some students finish this problem, let them find the two roads for 86 and the two roads for 92 using exactly three arrows of each color.

After about ten minutes, continue the collective discussion.

T: What are the ending numbers of your roads?

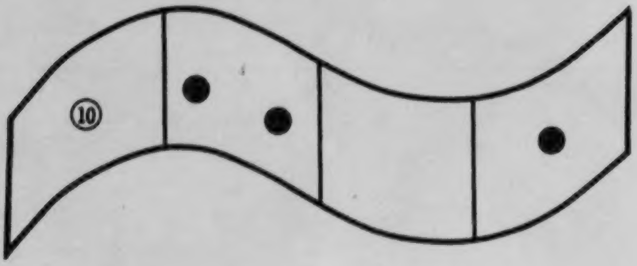
As students volunteer answers, list them on the board.

T: Do you see any connection between the Minicomputer clue and the arrow road clue?

Accept any reasonable response about the fact that the ending numbers of the roads are the same as the numbers in the Minicomputer clue. Those students that worked on the two-star problem may explain there are two roads ending at 90 just as there are two ways of putting 90 on the Minicomputer using one dime on the 8-square and three pennies.

Distribute the copies of "Clinton Street - Book A" and let a student read pages 28 and 29 aloud.

Draw this snake on the board. Here blue checkers are used for pennies because of their relation with the blue +1 arrows in the arrow roads.

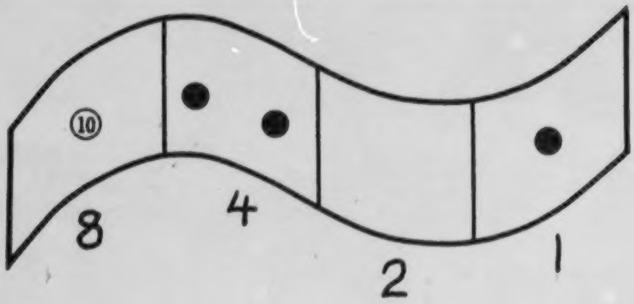


T: What do you think this number is?

Let a volunteer explain what number he or she thinks it is. In our development classes, one explanation went like this.

S: 89, because its like a Minicomputer in a different way.

The student then labeled the snake in this way.

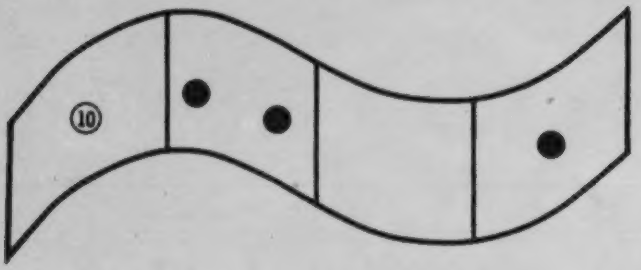


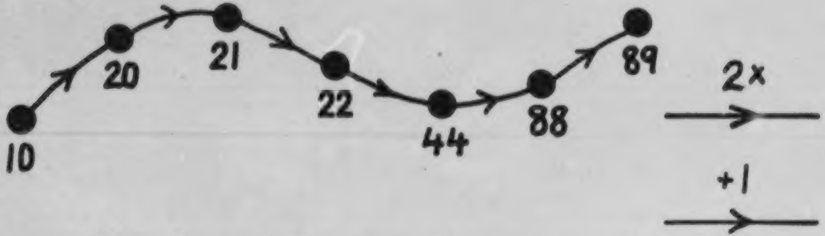
S: A dime on the 8-square, two pennies on the 4-square, and one penny on the 1-square makes 89.

T: Spike used an arrow road to solve the snake clue.

10W

Draw Spike's arrow road on the board, having students label the dots.





T: Why do you think Spike made this arrow picture to go with this snake?

S: The red and blue in the snake tells you what to do.

S: Start with 10, then double from one board to the next, then +1 for one penny, +1 for another penny, double, double, and finally +1 for the last penny.

At this point, let the class finish reading and do the work on page 32 of Book A. Distribute copies of "Clinton Street - Book B". Encourage everyone to complete pages 2 - 5 of Book B. The faster workers could study the remainder of the story-workbook if time permits.

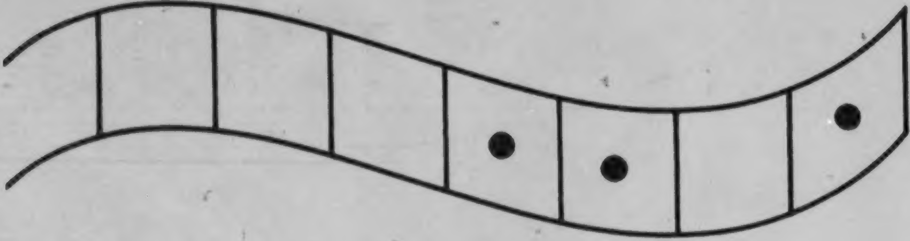
Don't forget to collect the story-workbooks at the end of the class.

Lesson 4

Do not distribute copies of "Clinton Street - Book B" until later.

Exercise 1

Draw this snake on the board.



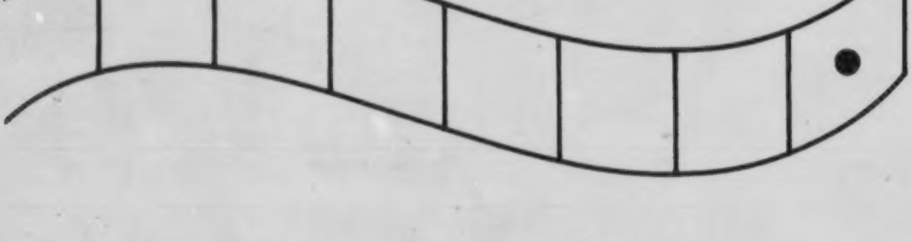
T: Do you remember the snake clue from "Clinton Street"? What is this number?

S: 13.

T: Why?

S: 1 on the first board, 4 on the third board, 8 on the fourth board. 1 + 4 + 8 = 13.

Put one checker on the first board.

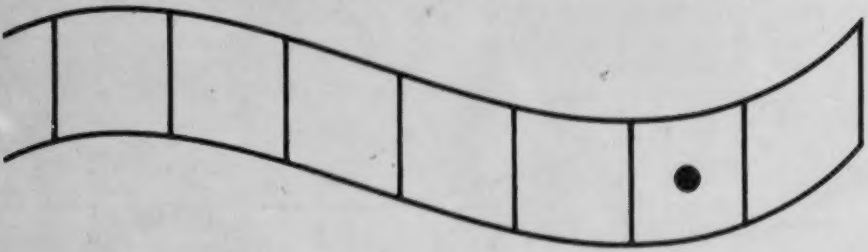


11W

T: What number is this?

S: 1.

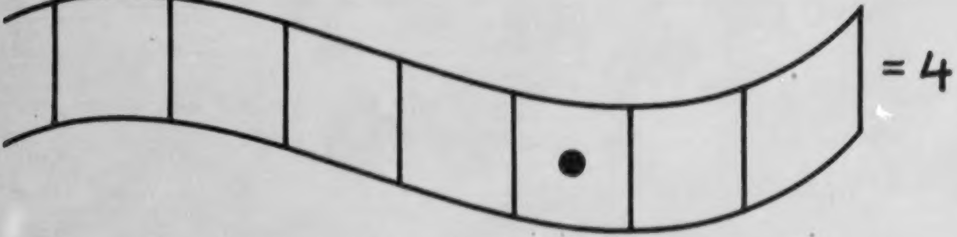
Move the checker one board to the left.

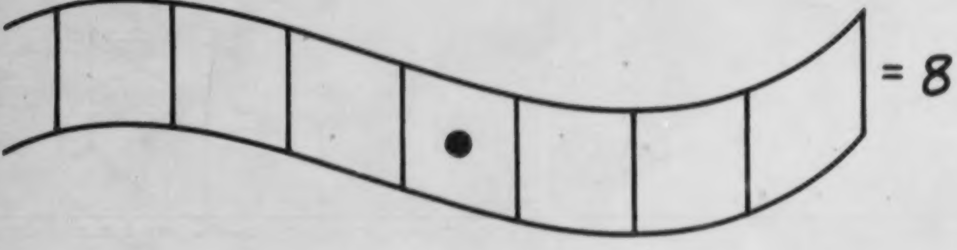


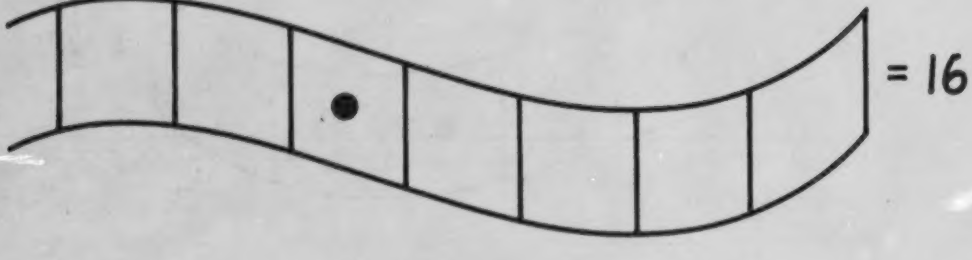
T: What number is this?

S: 2.

Continue moving the checker to the left and asking for the new number.





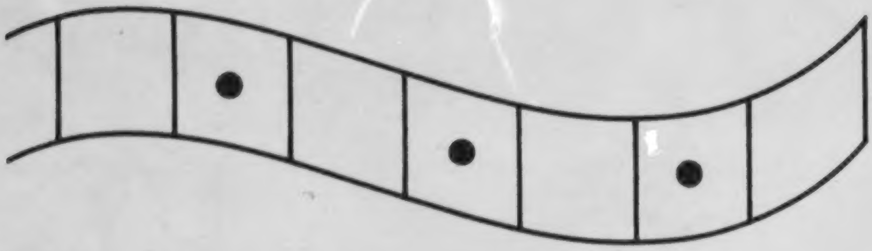


It is likely that some students will think this number is 10 rather than 16. Should this happen, remind them that the snake is different from the Minicomputer.

T: Look at the pattern of doubling; 1, 2, 4, 8, then ...

S: 16.

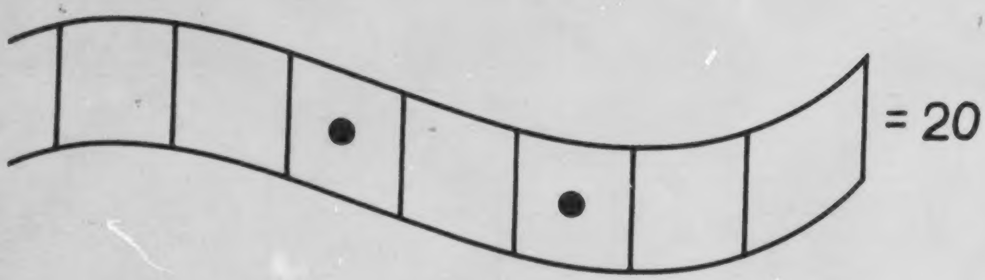
Continue in this manner until 1,024 is reached. Next, put this configuration on your snake.



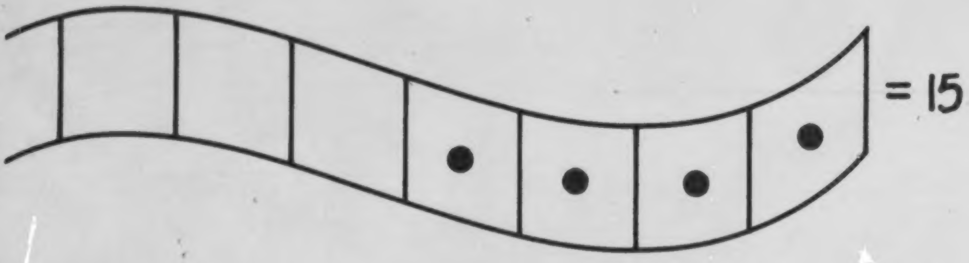
T: Write this number on your paper. (Answer: 42)

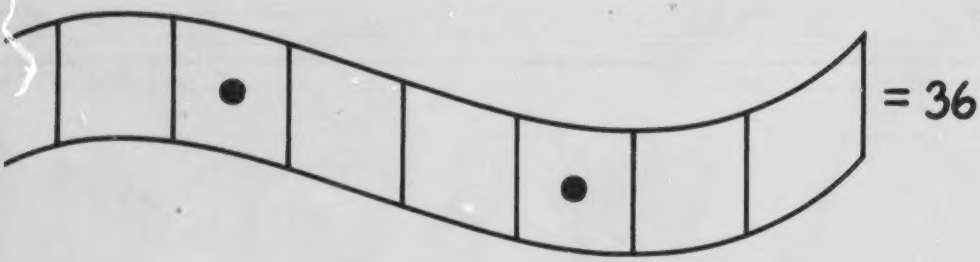
T: I would like someone to put 20 on the snake using at most one checker on each square.

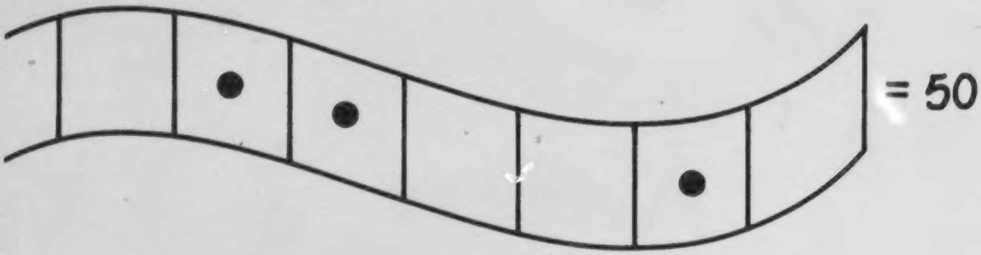
11W



Continue in this manner with 15, 36, and 50.







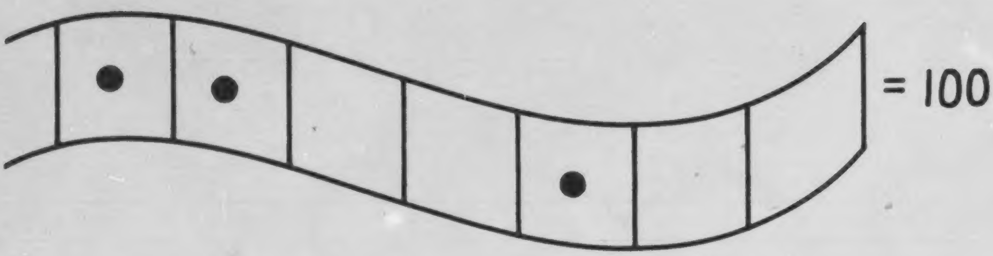
T: Here we have 50 on the snake. How can we get 100 immediately?

S: Add one checker to each square where there is already a checker; add another 50.

T: That's a very good idea, but I would like to have at most one checker on each square.

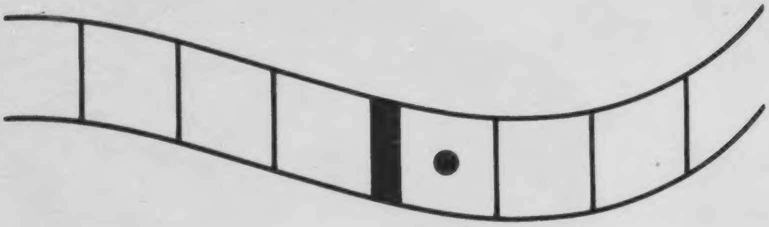
S: Move each checker over one square to the left because that will double each number.

S: Yes, 64 + 32 + 4 equals 100.



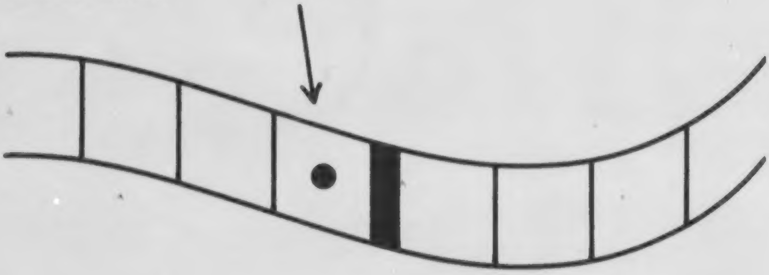
Exercise 2

Draw this snake.

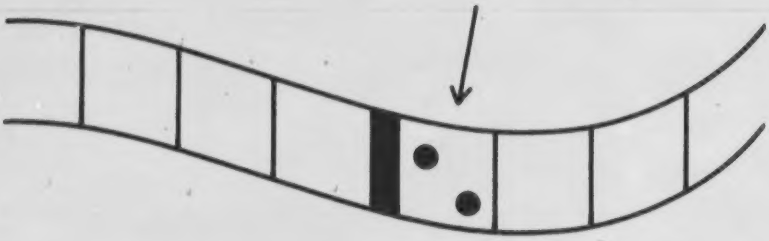


T: I have added a bar. What number do you think is on the snake now? Write this number on your paper.

Let the students work on this problem for a few minutes. Walk around and monitor student progress. It is likely that some students will write "0.8" or "0.1" or "0.2" as their answers to your question (the correct answer is $\frac{1}{2}$ or 0.5). After a few minutes, go to the board and remind them that one checker on this board

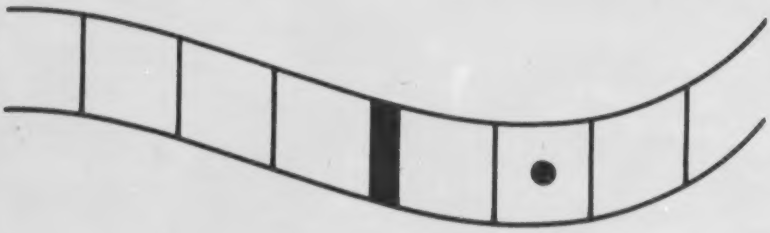


is the same number as two checkers on this board.

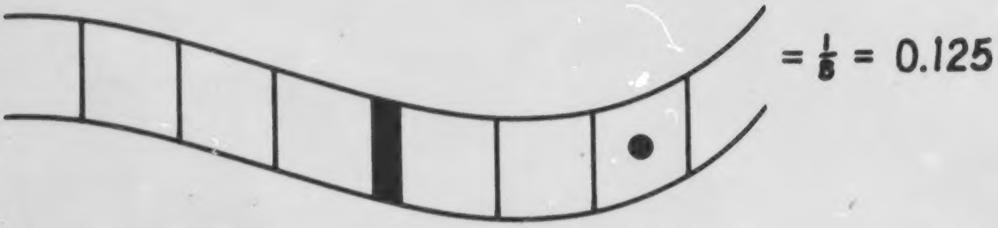


T: What is this number? (Answer: $\frac{1}{2}$ or 0.5)

Write this information on the board. Continue in this manner, moving the checker to the left.

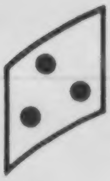


$= \frac{1}{4} = 0.25$



Exercise 3

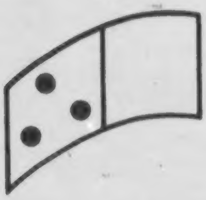
Draw a board with three checkers.



T: What number is this?

S: 3.

Add a board to the right.

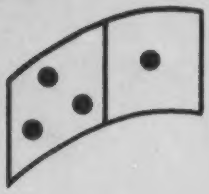


T: What number is this?

S: 6, because 3 x 2 = 6.

11W

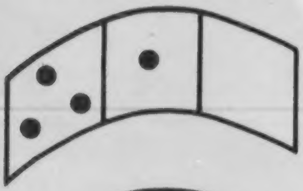
Add a checker.



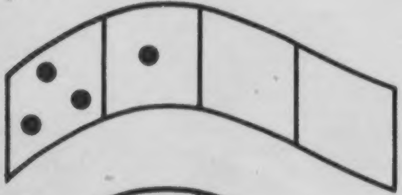
T: And this?

S: 7, because it's 6 + 1.

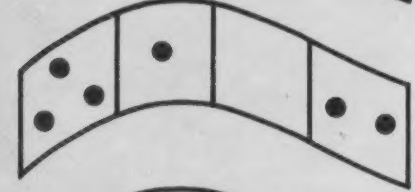
Continue in this manner slowly building a snake.



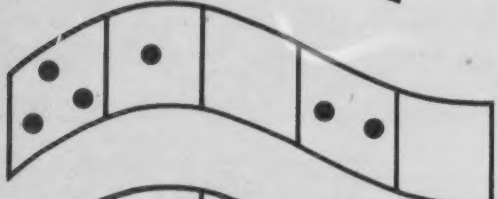
= 14



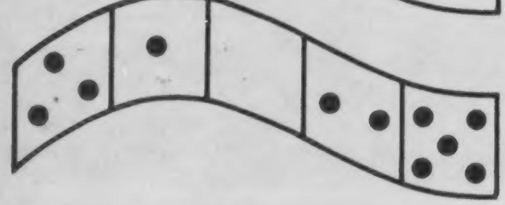
= 28



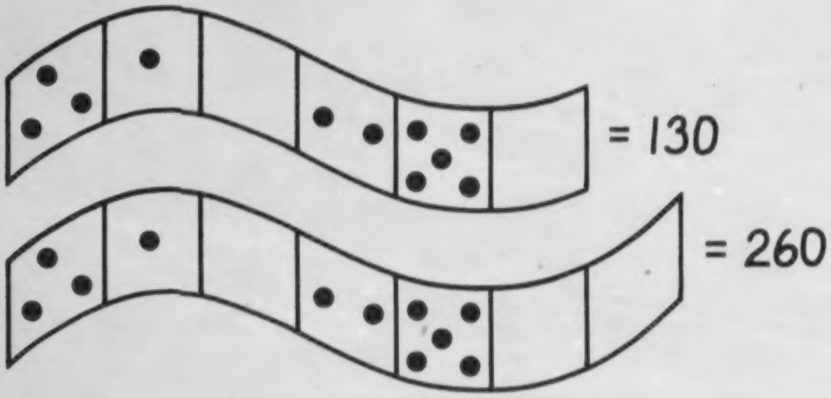
= 30



= 60



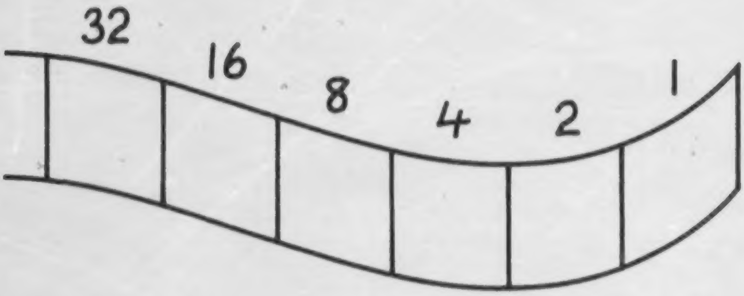
= 65



This exercise can be continued as long as there is student interest.

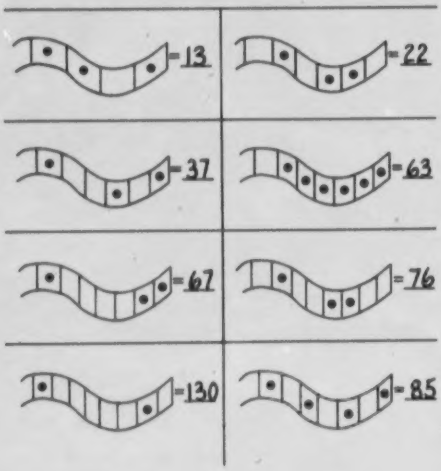
Distribute copies of "Clinton Street - Book A" and "Clinton Street - Book B" to the class. Have the students begin working on page 6 of Book B. They should work as far as they can. Everyone should complete through page 11 for the conclusion of the mystery. The faster students should be encouraged to complete Book B. The material beyond page 11 is an interesting extension of the abacus to other bases.

Worksheets N105, N106, and N107 are available for the remainder of the lesson. If any students have difficulty getting started with N105, encourage them to label each board of the snake with its value.



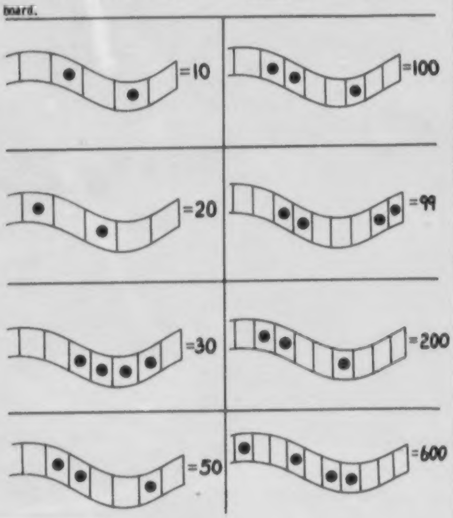
Name ____________________ **N 105 ***

ON THE BINARY SNAKE



Name ____________________ **N 106 ****

Put each number on the binary snake. Use at most one checker on each board.



Name ____________________ **N 107 *****

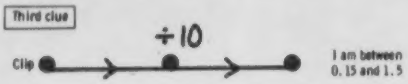
First clue Clip is a positive prime number.

Clip could be 2 ; 3 ; 5 ; 7 ; 11 ; 13 ; 17 ; 19 ; 23 ; 29 ; 31 ; 37 ; 41 ; 43 ; 47 ; 53 ; and so on.

Second clue Clip can be put on this binary snake using exactly two checkers (positive or negative).

Clip is 2 or 3 or 5 or 7 or 17 or 31 or 127.

Third clue



Clip is 17 or 31 or 127.

Fourth clue If you multiply Clip by himself, you will find a number larger than 1,000.

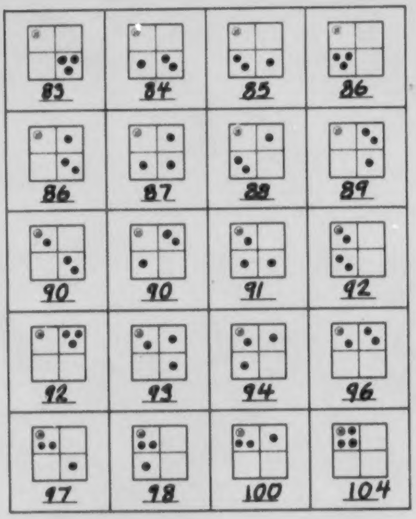
Clip is 127.

"It must be a clue to the number of the house in which the valuables are hidden. But it's a pity that the coins have fallen on the floor. There are so many possible numbers. I will have to search a lot of houses."

HOW MANY NUMBERS CAN YOU WRITE WITH A DIME AND THREE PENNIES ON A MINICOMPUTER BOARD? WRITE WHAT YOU THINK BELOW.

____

60 numbers can be written. Accept any estimate for this problem. A complete solution to this problem is given at the end of Lesson 1.



The black dots represent ① - checkers.

Spike found 20 different pictures, but only 17 different numbers. Three of them can be shown on a Minicomputer board in two ways. They are

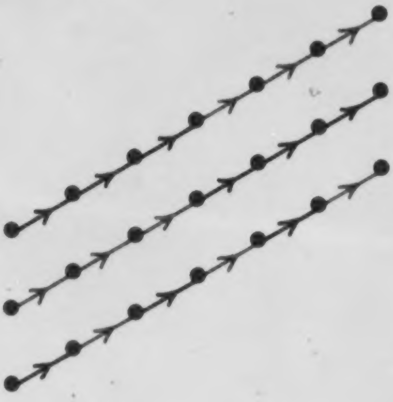
86, 90, and 92.

Only five numbers between 83 and 104 cannot be shown with a dime on the 8-square and three pennies somewhere or other on the Minicomputer board. They are

95, 99, 101, 102, and 103.

"I will have to search 17 houses," concluded Spike.

He drew some of the roads.



ON A SEPARATE SHEET OF PAPER, TRY TO DRAW ALL THE POSSIBLE ROADS. HOW MANY ARE THERE?

There are 20 roads.

YOU DO IT TOO ON THE ROADS THAT ARE DRAWN ON PAGES 22 AND 23. WHAT DO YOU NOTICE? *

DID YOU FIND 20 DIFFERENT NUMBERS?

No, only 17 different numbers.

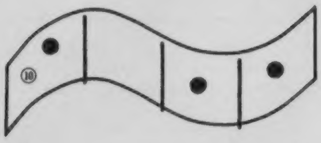
DO YOU SEE SOME CONNECTION BETWEEN THE ROADS CLUE AND THE MINICOMPUTER CLUE? IF SO, WHAT CONNECTION DO YOU SEE?

WRITE YOUR ANSWER BELOW.

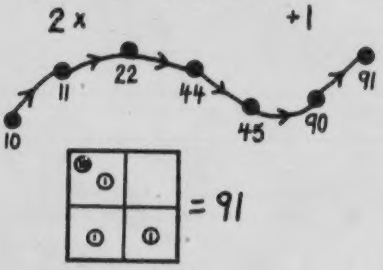
Any comment relating the ending numbers of the arrow road, starting at 10 and consisting of three red and three blue arrows, to the numbers that can be written on the Minicomputer using exactly one dime and three pennies should be accepted. The intention is to have students be aware that there is a connection, and not to describe this connection until later in the story of "Clinton Street".

* The ending numbers are the same as the numbers that can be put on the Minicomputer with a dime on the 8-square and three pennies somewhere or other on the Minicomputer.

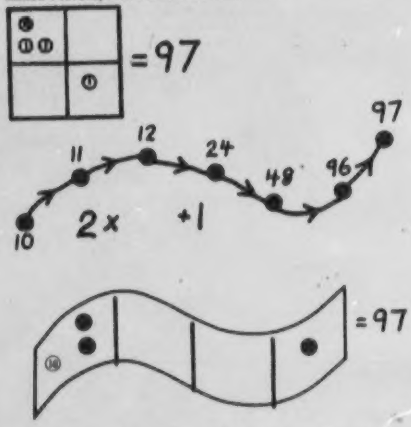
Spike moved the pennies around on the snake.



TRY TO DECODE THIS PICTURE BY DRAWING AN ARROW PICTURE ON THIS PAGE AND THEN BY DRAWING A MINICOMPUTER PICTURE BELOW.



PICK ANY NUMBER (EXCEPT 95, 99, 101, 102, 103) FROM 83 TO 104 AND, IN THE SPACE BELOW, DRAW A MINICOMPUTER PICTURE, AN ARROW PICTURE, AND A SNAKE PICTURE FOR IT.



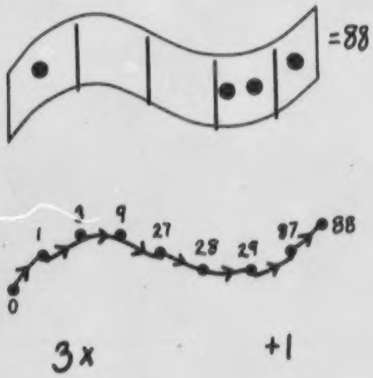
DO YOU THINK THAT THE SNAKE MESSAGE HELPED SPIKE IN HIS TREASURE HUNT?

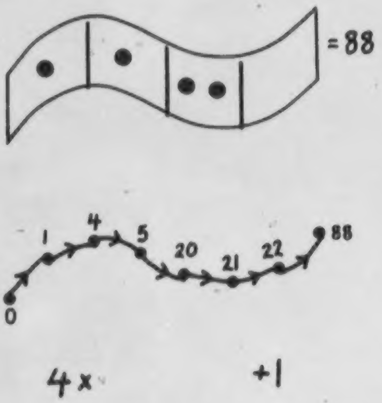
There are many possibilities.

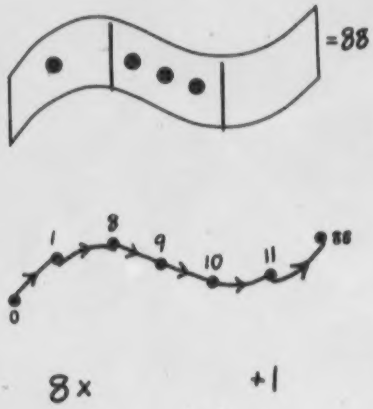
One is shown above.

J and Spike had a great time playing with snakes. They found three ways of showing 88 with exactly four pennies.

CHECK THEIR PICTURES ON PAGES 12, 13, AND 14 BY DECODING EACH ONE. LABEL THE DOTS IN EACH ARROW PICTURE.







A SET OF PROBLEMS NO. 3

A SET OF PROBLEMS NO. 3

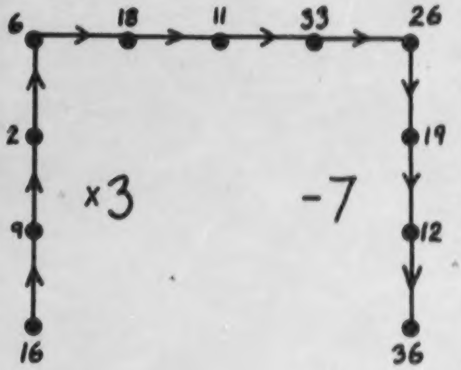
This workbook series has three levels of difficulty (* and ** in one booklet and *** in another booklet). It is suggested that all the students begin with level * and continue through as many levels as they can handle during the three lessons scheduled for this series. Not all students are expected to complete all three levels of this workbook series. Only the faster students will reach the most challenging level (***).

This series has been designed to encourage the students to work independently; therefore, no collective introductory lesson is necessary. Though there are many different types of problems, all of them will be familiar to the students because of previous work they have done in one or another of the strands.

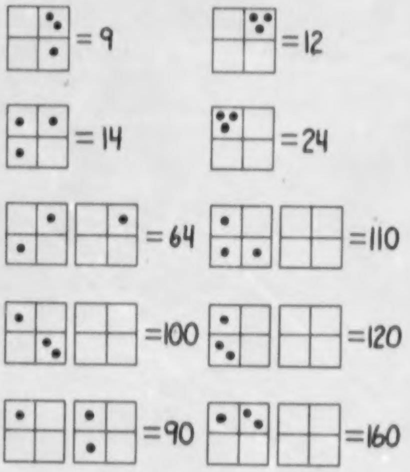
If you observe that many students are having difficulty with a particular page, you may wish to stop the independent work while you clarify the problem collectively. Then let them resume their independent work as you continue to observe and to give individual assistance.

A Set of Problems No. 3 * and **

Zig is the largest odd number and Zag is the smallest even number in this picture.
Label all the dots.
Who is Zig? 33
Who is Zag? 2

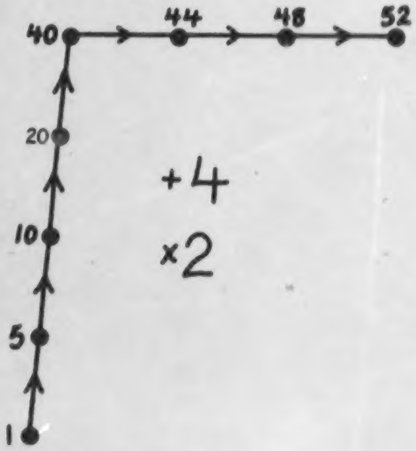


Write each of these numbers on the Minicomputer using exactly three positive checkers.

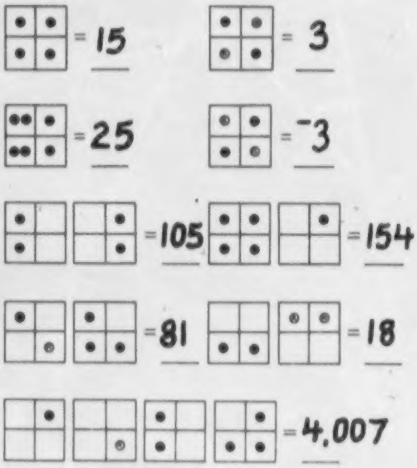


(Other solutions are possible.)

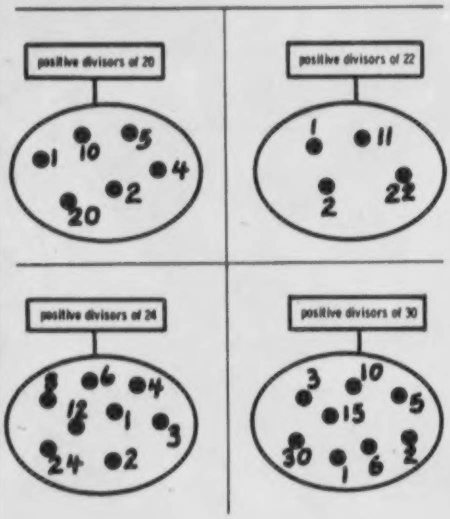
Dig is the only multiple of 3 in this arrow picture.
Label all the dots.
Who is Dig? 48



Complete:



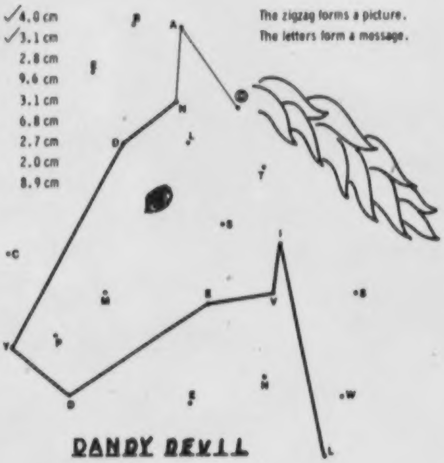
Label all the dots.



(The order in which the dots are labeled may vary.)

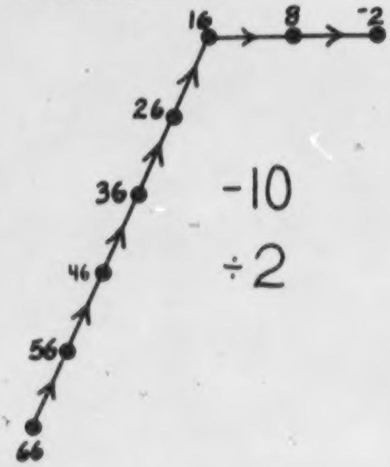
Follow the dots. Draw a zigzag starting at the dot marked "Ⓓ." The first two segments are drawn for you. Measure them. The first is 4.0 cm long. The second is 3.1 cm long. Use the rest of this list of lengths in order to complete the zigzag.

The zigzag forms a picture.
The letters form a message.



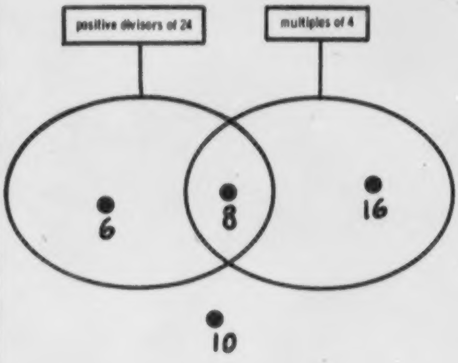
DANDY DEVIL

Dog is the only positive divisor of 24 in this arrow picture.
Label all the dots.
Who is Dog? 8



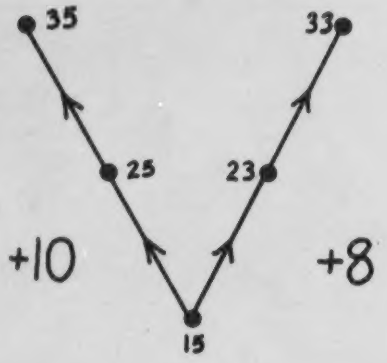
Label each dot below with one of these four numbers.

6; 8; 10; 16.



Label each dot below with one of these five numbers.

15; 23; 25; 33; 35.

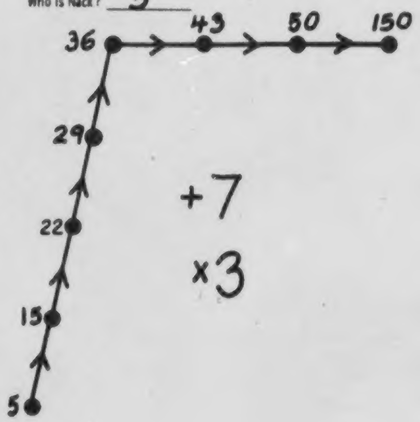


Nick is the only multiple of 4 and Nack is the only divisor of 20 in this arrow picture.

Label all the dots.

Who is Nick? 36

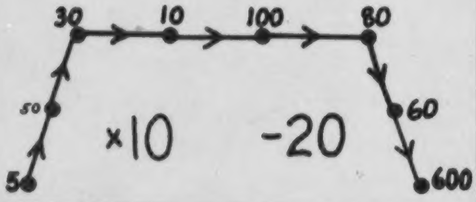
Who is Nack? 5

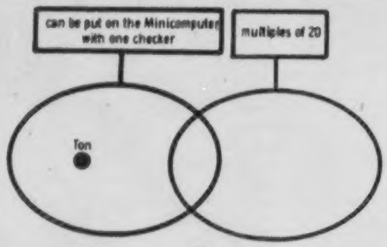


Ton is a secret number.

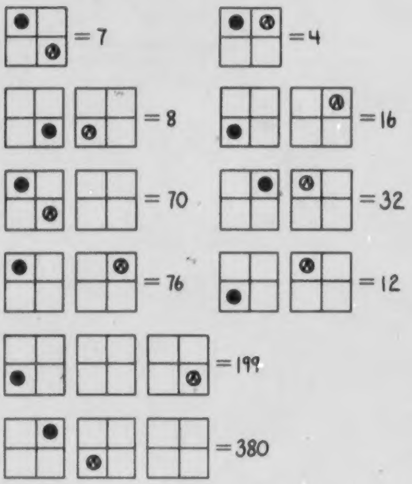
Ton is in this arrow picture and in this string picture.

Who is Ton? 10





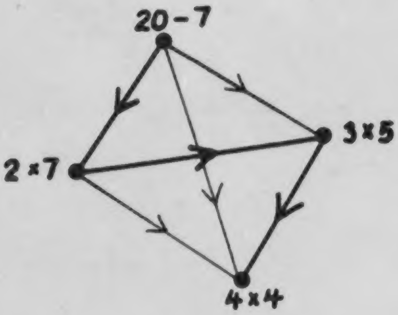
Put each of these numbers on the Minicomputer using exactly one positive and one negative checker.



Label each of the dots below with one of these four numbers.

3 x 5; 2 x 7; 4 x 4; 20 - 7.

Now draw all the missing arrows.

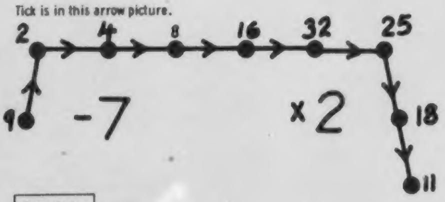


is smaller than

Tick is a secret number.

First clue

Tick is in this arrow picture.



Second clue

If you put Tick on the display of your hand-calculator and press $\boxed{+}\boxed{=}\boxed{=}$. . . , 100 will appear on the display.

Tick is 2 or 4 or 25.

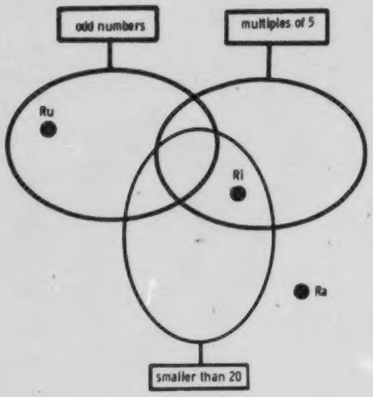
Third clue

Tick cannot be put on the Minicomputer with exactly one checker.

Tick is 25.

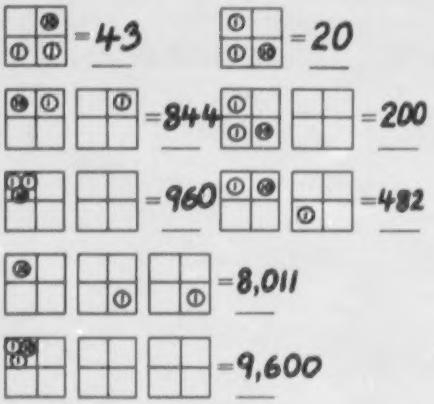
Ri, Ra and Ru are three of these numbers:

7; 8; 10; 15; 22; 25; 27; 30.



Ri is 10 ; Ra is 22 ; Ru is 27 .

Write any number you wish on the Minicomputer using exactly one dime and two pennies.
(Use "⑩" for the dime and "①" for each penny.)

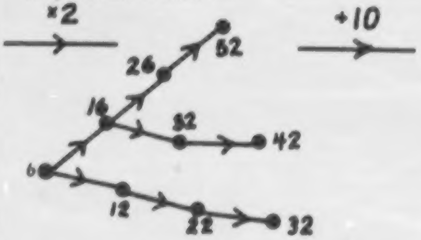


(Many other solutions are possible.)

Rig is a secret number.

First clue

Rig is the ending number of a red-green road starting at 6 and using two green arrows and one red arrow.



Rig is 52 or 42 or 32.

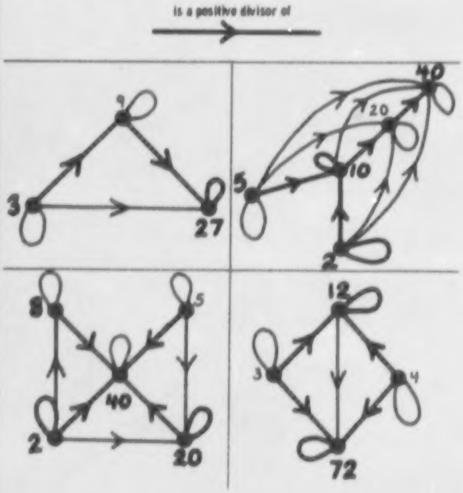
Second clue

Rig is not a multiple of 4.

Rig is 42.

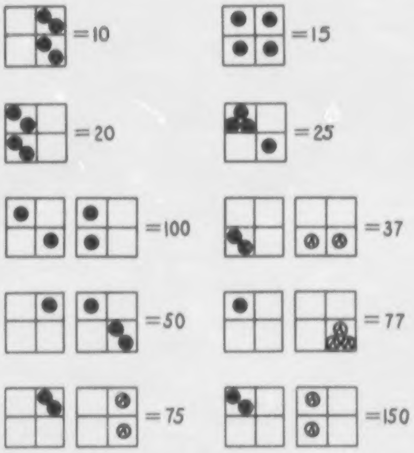
Note: There are two dots for 32 in this picture. The usual convention is suspended for this drawing for the sake of simplicity.

First label the dots. Several solutions are possible. Then draw the missing arrows.



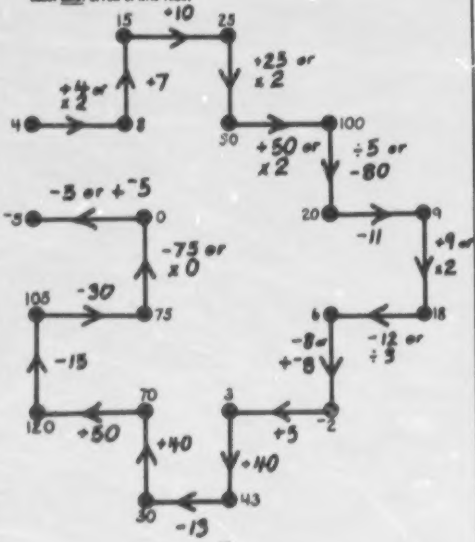
(Several solutions are possible.)

Write each of these numbers on the Minicomputer using exactly four checkers (positive or negative).



(Many solutions are possible.)

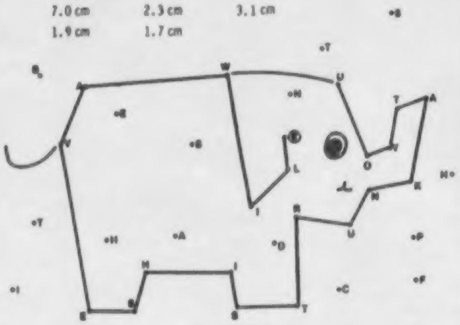
Label each arrow of this road.



(Several solutions are possible.)

Follow the dots. Draw a zigzag starting at the dot marked "Ⓔ."
Use this list of lengths in order for the segments of the zigzag. Complete the message at the bottom of the page.

1.3 cm
2.2 cm
5.5 cm
5.8 cm
2.6 cm
7.0 cm
1.9 cm
1.7 cm
3.5 cm
1.5 cm
2.5 cm
3.7 cm
2.3 cm
1.7 cm
1.7 cm
3.5 cm
1.3 cm
1.6 cm
1.0 cm
3.1 cm

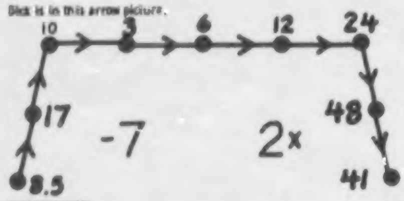


ELI WAVES HIS
TRUNK AT YOU

Dick is a secret number.

First clue

Dick is in this arrow picture.



Second clue

Dick is a prime number.

Dick is 17 or 3 or 41.

Third clue

Dick cannot be put on the Minicomputer with exactly two checkers.

Dick is 17.

Write below the 13 numbers that you can put on the ones' board of the Minicomputer using exactly one positive checker and one negative checker.

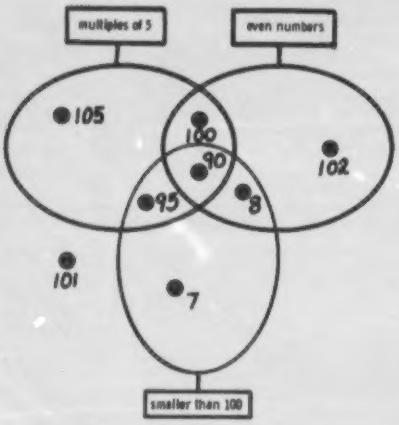
0 ; 4 ; 6 ; 7 ; ⁻4 ; 2 ; 3 ; ⁻6 ; ⁻2 ; 1 ; ⁻7 ; ⁻3 ; ⁻1 .

If you add all these numbers together and then double the result, you will get Zu.

Zu is 0.

Label each dot in the string picture below with one of these numbers.

7; 8; 90; 95; 100; 101; 102; 105.

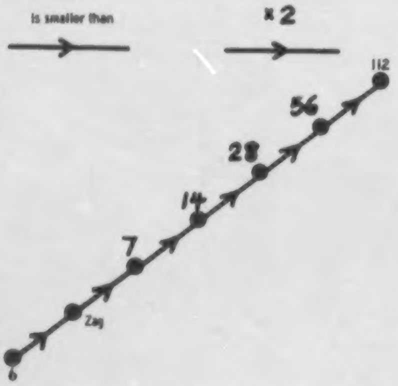


Write the four prime numbers that are between 20 and 40.

23; 29; 31; 37.

If you add all these numbers together and then divide the result by 100, you will get Zut.

Zut is 1.2.



Write at least ten numbers that Zag could be.

6.01; 6.05; 6.1; 6.25; 6.3;

6.4; 6.5; 6.67; 6.83; 6.99;

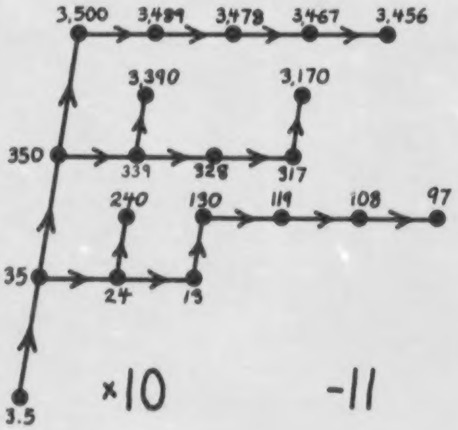
and so on.

(Any number between 6 and 7 is a solution.)

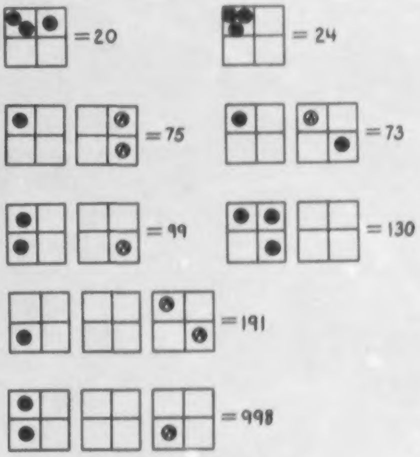
Label all the dots.

Fi is the smallest multiple of 10 in this arrow picture.

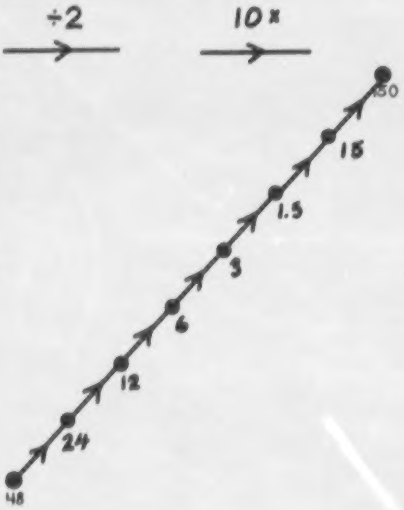
Who is Fi? 130



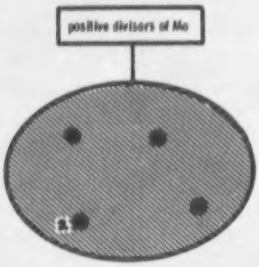
Write each of these numbers on the Minicomputer using exactly three checkers (positive or negative).



Build a red-green road from 48 to 150.



(Several solutions are possible.)



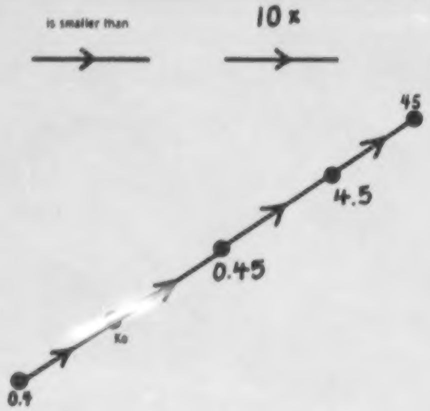
Write at least five numbers that Mo could be.

6; 8; 10; 14; 22;

and so on.

(Many other solutions are possible.)

A Set of Problems No. 3 ***



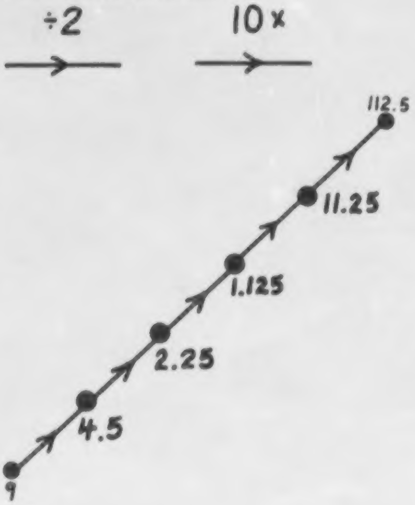
Ko could be 0.41; 0.42; 0.425; 0.43; 0.435;

0.438; 0.442; 0.445; 0.448; 0.4411; and so on.

2

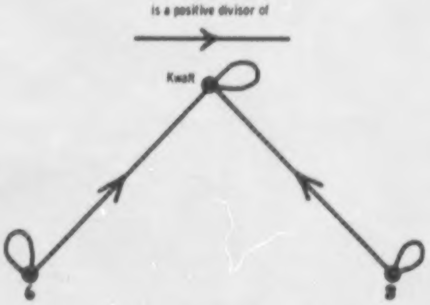
(Any number between 0.4 and 0.45
is a solution.)

Build a red-green road from 9 to 112.5.



3

(Other solutions are possible.)



Kwaft could be 24; 48; 72; 96; 120;

144; 168; 192; 216; 240; and so on.

Which is the smallest number that Kwaft could be? 24

What do you notice about the numbers that Kwaft could be? Write your answer below.

The numbers that Kwaft could be are

positive multiples of 24.

4

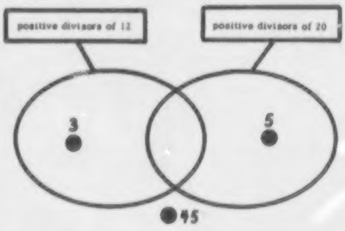
The red tag is one of these:

- ~~Positive prime numbers~~
- ~~Multiples of [illegible]~~
- ~~Multiples of [illegible]~~
- Positive divisors of 12
- ~~Positive divisors of 20~~
- ~~Larger than [illegible]~~
- ~~Smaller than [illegible]~~
- ~~Odd numbers~~

The green tag is one of these:

- ~~Positive prime numbers~~
- ~~Multiples of [illegible]~~
- ~~Multiples of [illegible]~~
- ~~Positive divisors of 12~~
- Positive divisors of 20
- ~~Larger than [illegible]~~
- ~~Smaller than [illegible]~~
- ~~Odd numbers~~

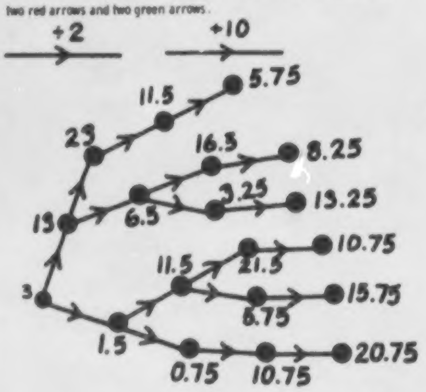
Label the strings.



5

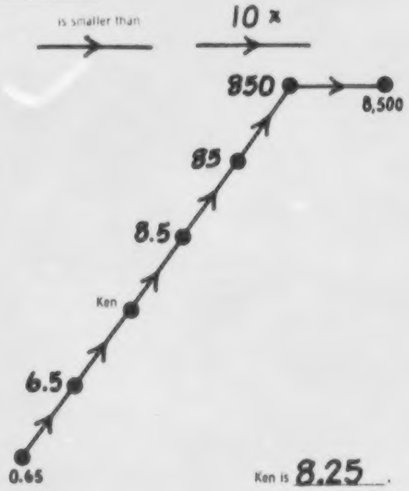
First clue

Ken is the ending number of a red-green road starting at 3 and using two red arrows and two green arrows.



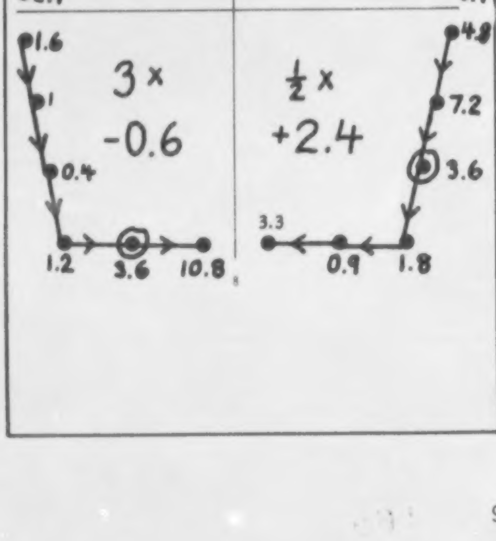
Ken is 20.75 or 15.75 or 10.75 or 13.25 or 8.25 or 5.75.

Second clue

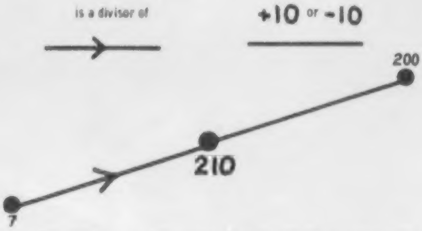


Ken is 8.25.

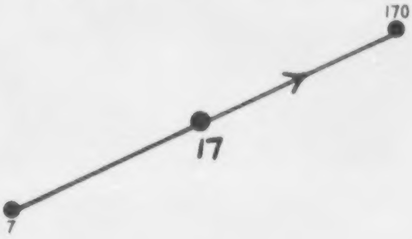
3.6 is in each of these pictures. In each one, circle its dot.



Try to build the shortest green-red road from 7 to 200.

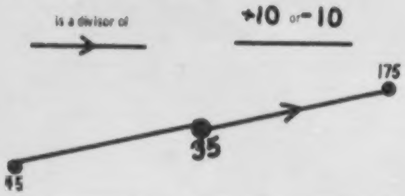


Do a similar exercise from 7 to 170.

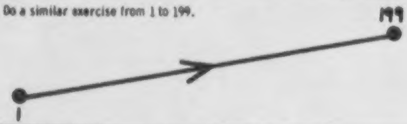


(Note: Other roads are possible for both of these problems but all of them will be longer than these.)

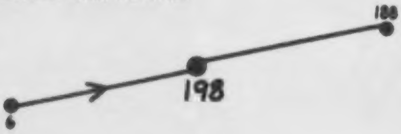
Try to build the shortest green-red road from 45 to 175.



Do a similar exercise from 1 to 199.



Do a similar exercise from 6 to 188.



10

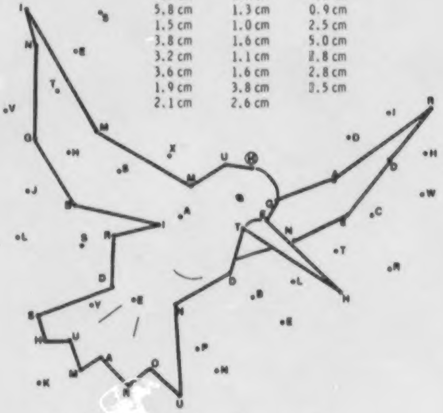
(Note: Other roads are possible for each of these problems but all of them are longer than these.)

Follow the dots. Draw a zigzag starting at the dot marked Ⓗ. Use this list of lengths (in order) for the segments of the zigzag. Complete the message at the bottom of the page.

1.1 cm
1.7 cm
4.5 cm
5.8 cm
1.5 cm
3.8 cm
3.2 cm
3.6 cm
1.9 cm
2.1 cm

3.3 cm
1.1 cm
1.0 cm
1.3 cm
1.0 cm
1.6 cm
1.1 cm
1.6 cm
3.8 cm
2.6 cm

1.9 cm
4.8 cm
4.2 cm
0.9 cm
2.5 cm
5.0 cm
2.8 cm
2.8 cm
3.5 cm



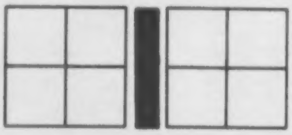
HUMM[illegible]GBIRDS HUM
AROUN[illegible] THE GARDEN

11

Kwan is a secret number.

First clue

Kwan can be put on the Minicomputer using exactly one positive checker on the ones' board and one negative checker on the tenths' board.

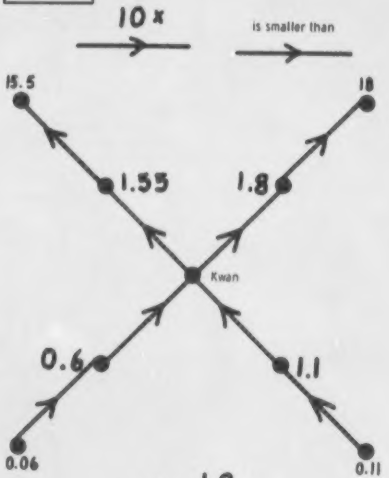


Write below the 16 numbers that Kwan could be.

0.2, 0.6, 0.8, 0.9,
1.2, 1.6, 1.8, 1.9,
3.2, 3.6, 3.8, 3.9,
7.2, 7.6, 7.8, 7.9.

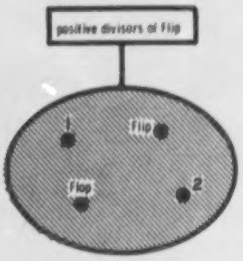
12

Second clue



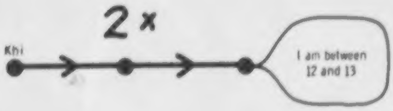
Kwan is 1.2.

13



Fill in this table to show some numbers that Flop and Flip could be.

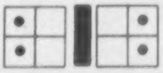
Flop	3	4	5	7	11	13	19	23	and so on.
Flip	6	8	10	14	22	26	38	46	

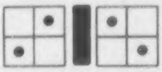


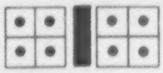
Khi could be 3.01; 3.05; 3.09; 3.1; 3.15; 3.19; 3.2; 3.21; 3.24; 3.249; and so on.

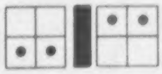
(Any number between 3 and 3.25 is a solution.)

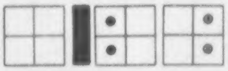
Complete:

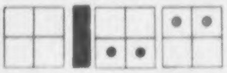

= 9.5


= 5.1


= 13.5


= 1.8


= 0.95


= 0.18

172

NABU WINS AN AWARD

NABU WINS AN AWARD

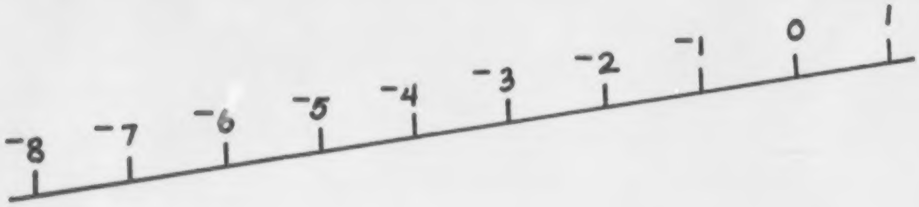
The purpose of this lesson is to explore certain properties of the number system in a very open way; a way that should provide an opportunity for students to participate at their level of understanding. It will be up to you, the teacher, to direct the discussion and to capitalize on insights that students might have.

Below are some questions and student comments that were made during the presentation of this story to our development classes. Feel free to modify our presentation in any way that you see fit. Above all let the students explore and enjoy this story of Nabu.

Distribute a copy of the storybook NABU WINS AN AWARD to each student and have the story read aloud by students (or read it to your class).

Page 5

Draw this part of the number line on the board.



15W

T: Where is $^{-}\frac{1}{2}$ on the number line?

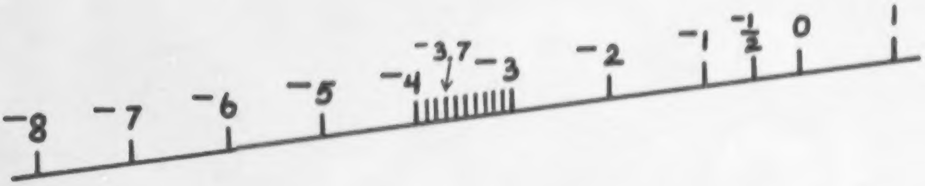
S: $^{-}\frac{1}{2}$ is halfway between $^{-}1$ and 0.

Have a student locate $^{-}\frac{1}{2}$ on the number line.

T: Where is $^{-}3.7$ on the number line?

S: You should divide the segment between $^{-}3$ and $^{-}4$ into ten smaller segments of equal length.

Do this and have a student locate $^{-}3.7$.



Pages 7 and 8

T: What do you think of the $\frac{1}{2}\times$ and $2\times$ dances?

S: It's the same dance, but $\frac{1}{2}\times$ goes one way and $2\times$ goes the other way.

T: Yes. The $\frac{1}{2}\times$ dance is just the converse of the $2\times$ dance.

S: The dances will go on forever. There is no beginning or ending number.

Pages 11 and 12

T: What about the -2 and +2 dances?

S: There are no odd numbers in these dances.

Note: Of course, these dances could have been made with just the odd numbers.

S: They are like the $\frac{1}{2}\times$ and $2\times$ dances - one goes in the opposite direction from the other.

T: Yes, they are converses of each other.

Page 15

T: If 15 were in this dance, who would be its partner?

S: $\frac{1}{15}$.

T: Why?

S: $15 \times \frac{1}{15} = 1 = \frac{1}{15} \times 15$.

Page 16

S: The dances look like flowers.

S: The + dance looks just like the × dance, but each positive number dances with a negative number.

T: Any negative number?

S: No, its opposite: 8 and $^{-}8$; $^{-}26$ and 26; and so on.

Page 19

S: This dance looks like teardrops.

S: It's raining numbers.

S: Each number dances with itself.

T: Why?

S: Because any number plus zero equals that number, and any number times one equals that number.

The ideal situation for the conclusion of this lesson is for the students to have their own copies of NABU WINS AN AWARD and to be allowed to keep them. If this is not feasible, a few copies should be placed in the class reading corner in order that students may reread the story at their leisure.

Play the string game for the rest of the period.

A STRANGE COUNTRY

A STRANGE COUNTRY

Lesson 1

Exercise 1

Distribute copies of the story-workbook " A Strange Country". Read pages 5-7 together and encourage the class to discuss the rule that Zero could have invented. Ask the students to continue reading (pages 8 and 9) and then to solve the problems on page 9. As you are checking the work, ask the students to give reasons for their answers.

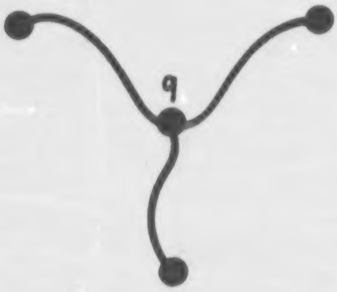
T: Are 12 and 25 directly connected? (Yes) Why?

S: 12 × 2 = 24 and 24 + 1 = 25.

T: Good. What about 7 and 5? (No) Why?

S: The double of 5 is 10 and the double plus one is 11.

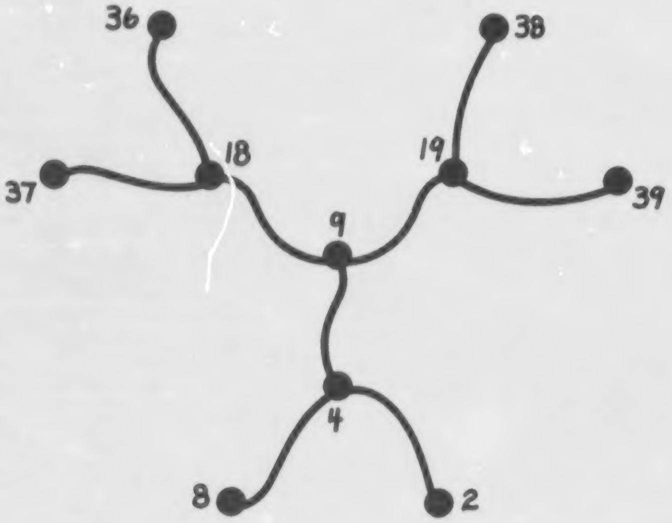
Have the students close their books for a while. At the board, slowly draw the neighborhood for 9.



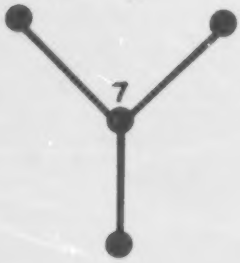
T: Who are the next neighbors of 9?

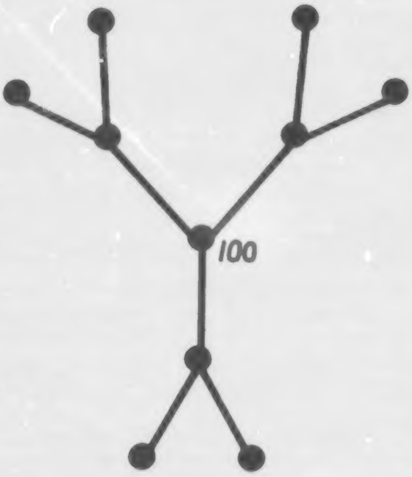
16W

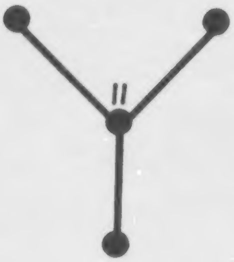
Let volunteers give answers and label the dots. Then expand the neighborhood and label the new neighbors.



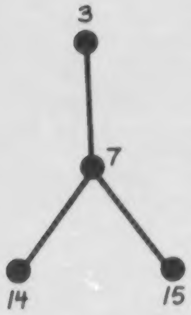
Write these problems on the board, and ask each student to copy the drawings and label the dots.

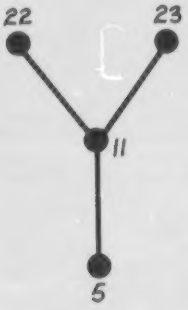


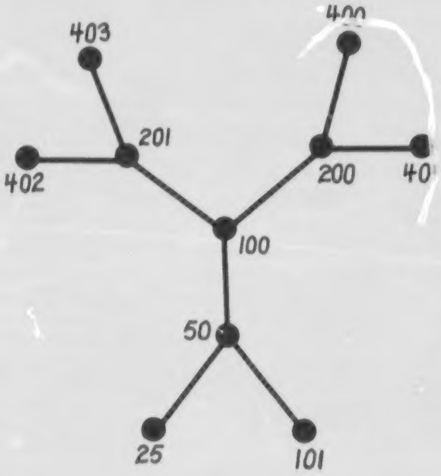




Should some students finish quickly, encourage them to expand their drawings and label the dots. When most have finished, check the work collectively. Answers are given below.

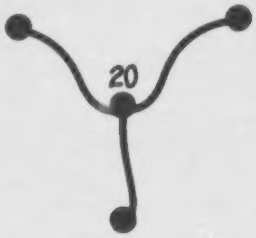






Let the students look at pages 10 and 11, and then read pages 12 and 13 together. Work on this problem collectively at first.

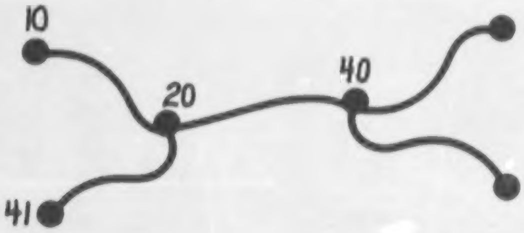
Draw this neighborhood of 20 on the board.



T: Who are 20's neighbors?

S: 40, 41, and 10.

Put these numbers in your picture and expand one branch.



T: One of 40's neighbors is 20. What are the other two?

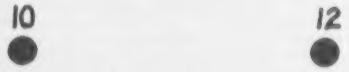
S: 80 and 81.

Label the two dots "80" and "81". Ask the students to close their books and to put them aside. Give each student a large sheet of paper, and ask them to copy the drawing on the board and to continue expanding the neighborhood. Allow at least 15 minutes for this activity.

If there is still some time available use this exercise.

Exercise 2

Draw these dots.

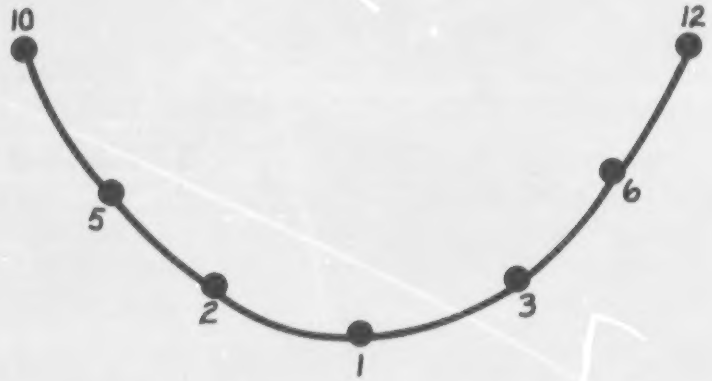


T: Are 10 and 12 connected directly? (No) Why?

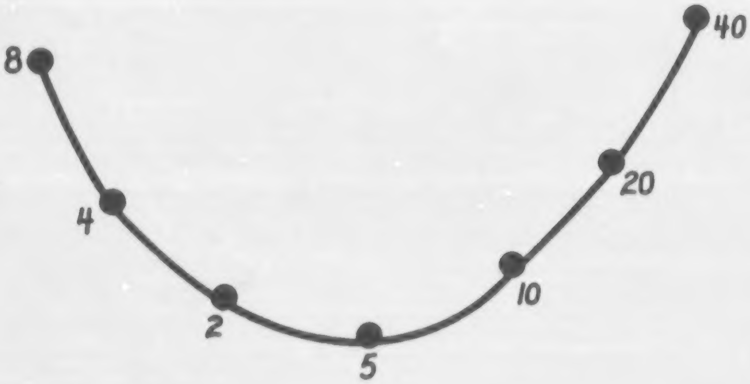
S: 2 × 10 = 20 and 20 + 1 = 21.

T: Okay, but 10 wanted to walk to 12's house. Can you find a road from 10 to 12?

Let the students solve this problem on their own. When someone finds the solution, have it drawn on the board. The solution is shown below.



Do a similar exercise from 8 to 40.



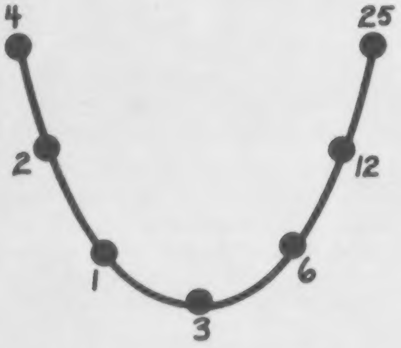
Finally let the students work on page 15. Should some students finish quickly, ask them to build roads from

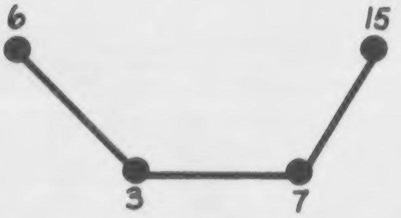
4 to 25;
6 to 15;
25 to 27;
46 to 50;
25 to 1,000.

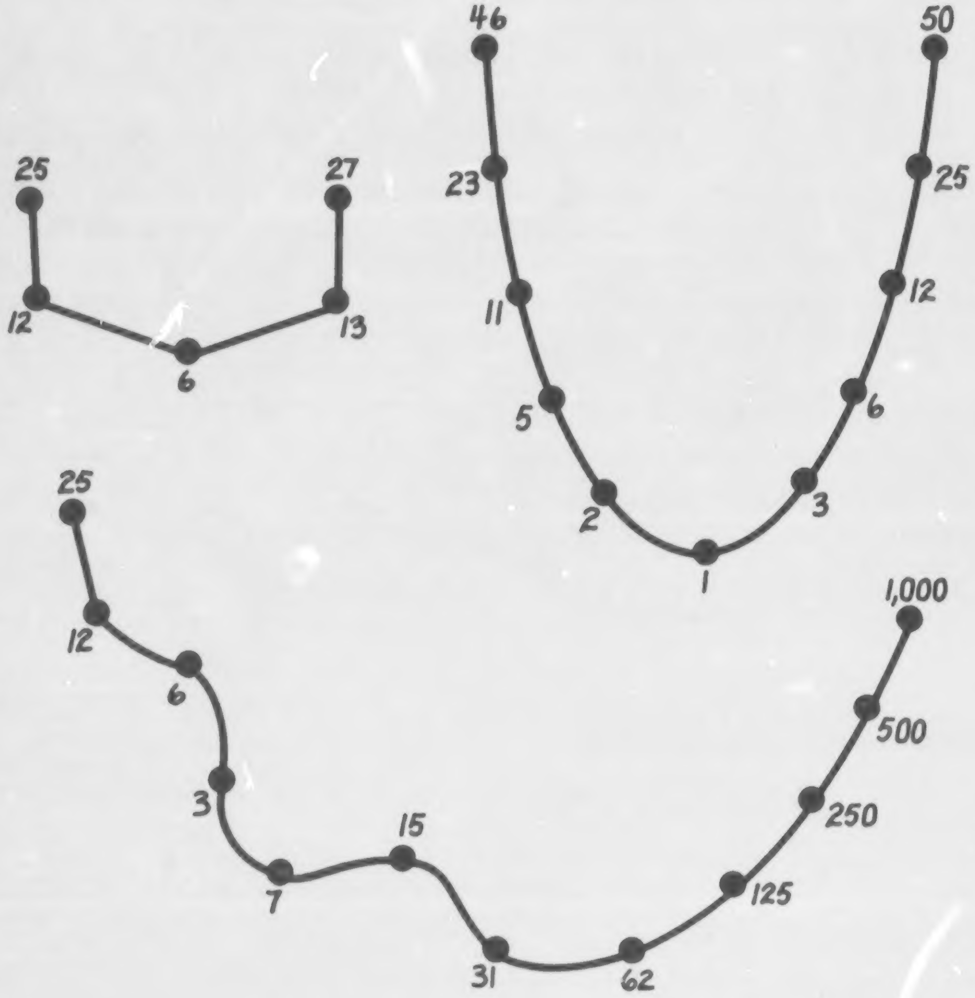
Solutions are given below.

183

16W







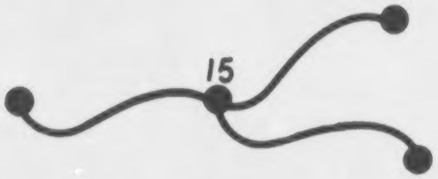
Be sure to collect the story-workbooks, as they will be used again.

Lesson 2

Do not distribute the story-workbook, "A Strange Country" for the moment. Instead, let a student tell the story of "A Strange Country" and remind the students of the rule that Zero invented.

Exercise 1

Draw this picture on the board.



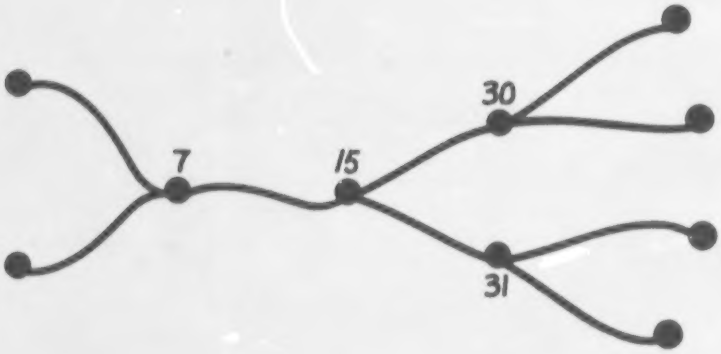
T: Which numbers are directly connected to 15? (31, 30, and 7) Why?

S: 30 = 2 × 15.

S: 31 = (2 × 15) + 1.

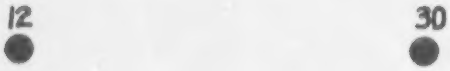
S: 15 = (2 × 7) + 1.

Label the dots as these responses are given; then add cords to your picture as shown in the next illustration. Ask the students to copy the drawing and label the dots. As they complete this drawing, encourage them to enlarge their drawings. At the same time, let some students label the dots you have drawn on the board.



Exercise 2

Draw two dots on the board and label one "12" and the other "30".

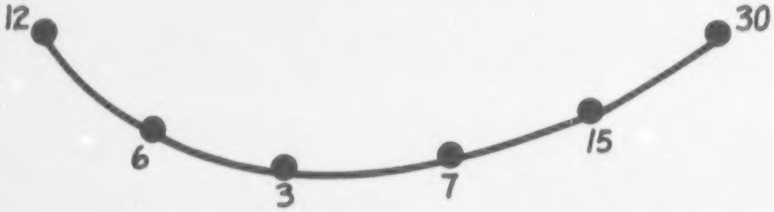


T: Are 12 and 30 directly connected? (No) Why?

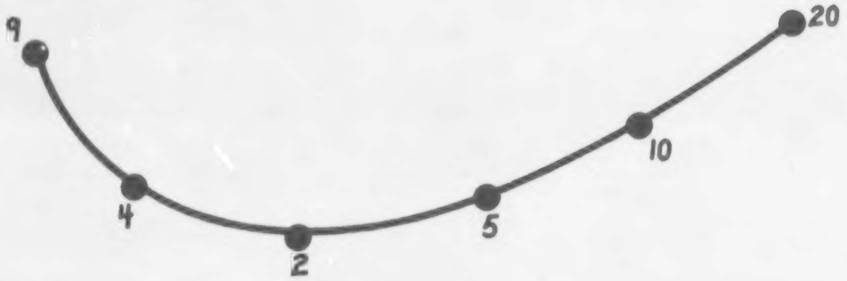
S: 2 × 12 = 24 and the double of 12 plus one is 25. Neither of these numbers is 30.

T: Good. Try to build a road between 12 and 30. Draw the road on your paper.

Let the students work on this for a few minutes. When a solution is found, ask that it be drawn on the board. The solution is shown below.



Do a similar exercise with 9 and 20.



Distribute the story-workbooks and have the students work independently on the problems on pages 17, 19, 21, and 23.

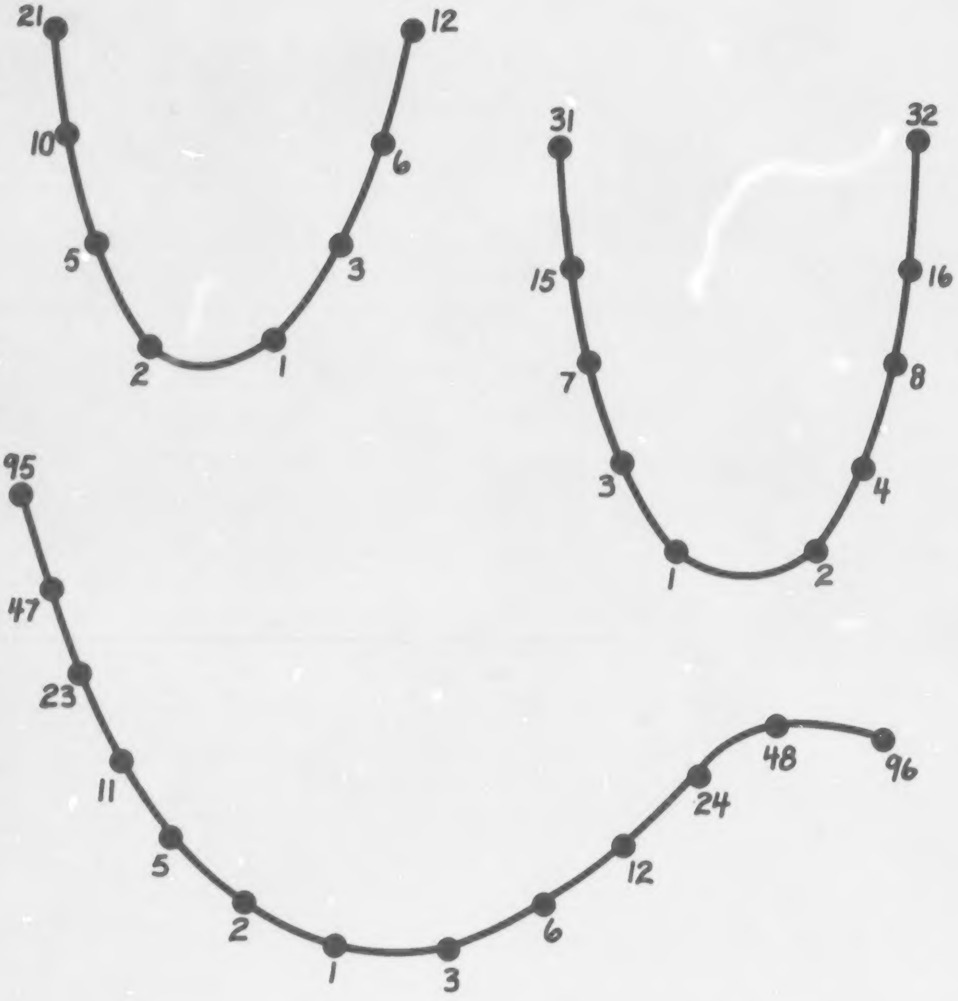
Students who finish these pages may be given the following problems to solve.

Build a road:

$\boxed{*}$ between 21 and 12.
$\boxed{**}$ between 31 and 32.
$\boxed{***}$ between 95 and 96.

The solutions are given on the next page.

17 W



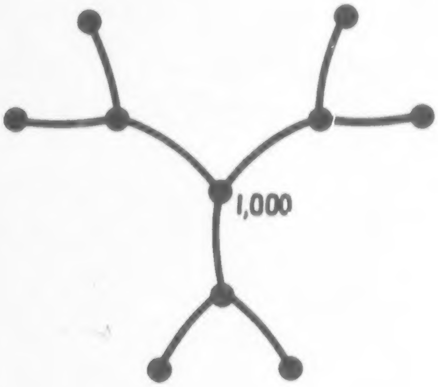
Be sure to collect the story-workbooks at the end of class, as they will be used again.

Lesson 3

Do not distribute the story-workbook, "A Strange Country", until later in the lesson. Give each student two large sheets of paper. Ask a student to remind the class of the rule for connecting the houses of two numbers.

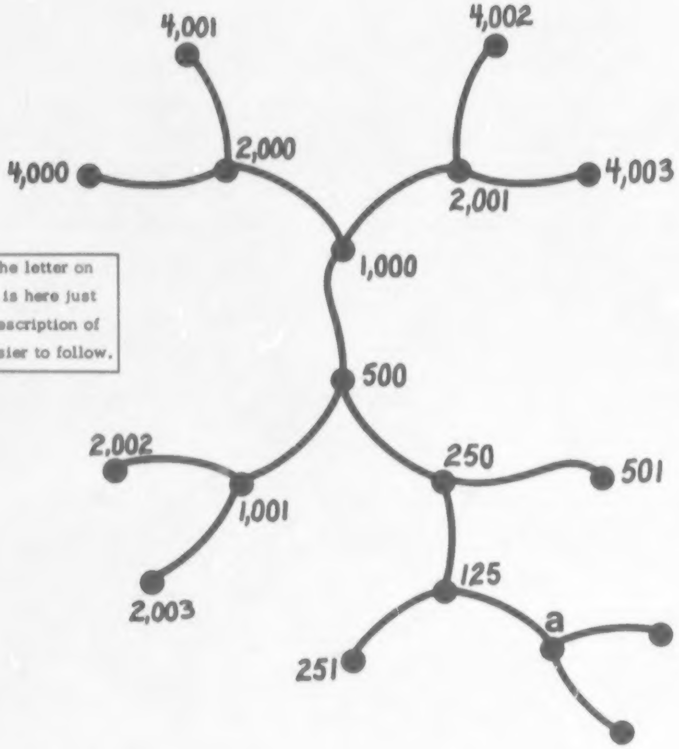
Exercise 1

Draw this picture (from page 23 of "A Strange Country") on the board.



T: Copy this drawing of 1,000's neighborhood on your paper. We will label these dots on the board, and while we are doing that you may expand the neighborhood on your paper.

While students are working by themselves, ask someone to come to the board and label two dots. This should provide you with a good opportunity to help some students who are having trouble getting started. Expand your picture and continue to let students label dots. When your picture looks something like this, Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.



initiate some class discussion.

T: Who is this next neighbor (point to a) of 125?

S: 62.

T: How did you decide on 62?

S: I subtracted 1 from 125 and halved 124.

S: (2 × 62) + 1 = 125.

T: Look at the picture you have drawn. You probably have different numbers in your picture. Tell me one number that is in your picture and is larger than any of the numbers on the board.

S: 8,000.

T: Does anyone have a number greater than 8,000 in their picture?

S: 16,014.

Continue this activity as long as there is student interest.

T: Tell me a number that is in your picture and smaller than any of the numbers on the board.

S: 31.

T: Does anyone have a number less than 31?

S: 3.

S: 1.

S: 0.

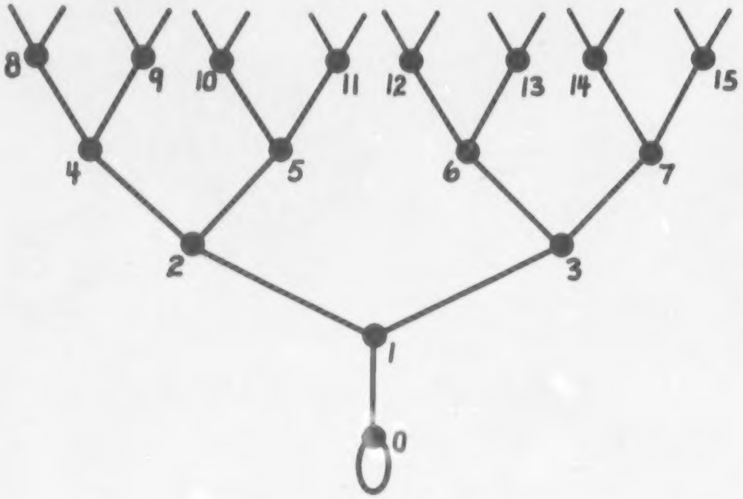
T: Would you agree that every whole number is directly connected to three other whole numbers?

S: No, 0 is only connected to 1 and to itself.

T: Very good. Let's take a look at 0's neighborhood. Draw on your paper 0's neighborhood and expand it. I will start a picture on the board.

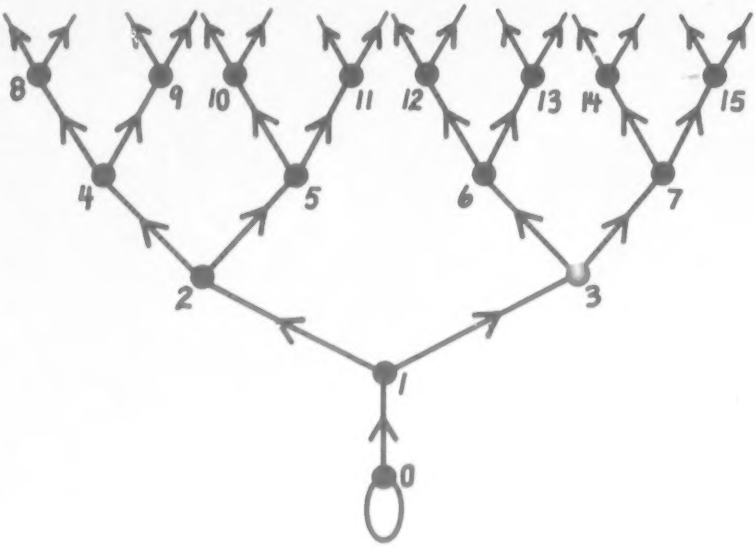
Exercise 2

After students have begun working on their drawings, draw this picture on the board and collectively label the dots.



T: Zero decided to add arrows to his picture to show the direction he would walk away from his house.

Add arrows to your picture.



T: Does this picture remind you of something?

S: A tree.

T: Very good. Do you remember what a function is?

S: Each dot can have at most one arrow starting from it.

T: Good. Is this relation (point to the tree) a function?

S: No. There are guilty points.

T: Come to the board and show me a guilty point.

Let several students show guilty points.

T: Are there any points that are not guilty?

S: 0.

S: No, there are two arrows starting at 0.

S: The loop is an arrow, and there is another arrow ending at 1.

T: What if I were to reverse the direction of each arrow in this picture? What could you say about the converse relation?

S: It's a function - exactly one arrow starts at each dot.

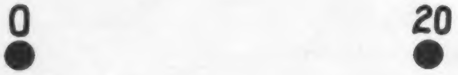
T: Before we go on to another exercise, do you notice anything else about this arrow picture (point to the picture on the board)?

S: Exactly one arrow ends at each dot and exactly two arrows start at each dot.

T: Good. Keep that idea in mind as it will come in handy a little bit later.

Exercise 3

Draw two dots on the board and label them "0" and "20".



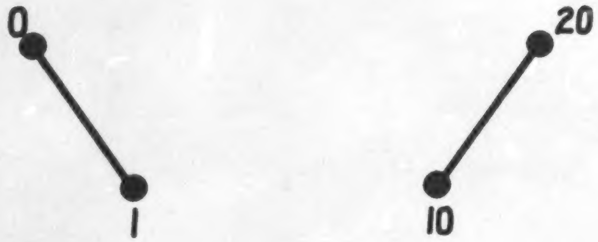
T: Let's build a road between 0 and 20. How could we start?

S: 0 to 1.

S: 20 to 10.

T: Good. I'll use both suggestions.

Add this information to your drawing.



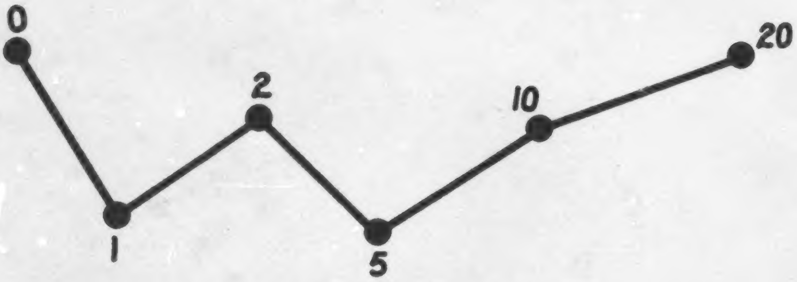
T: What next?

S: 10 to 5.

T: OK, then what?

S: I have it! 5 to 2 to 1.

Complete the road.



Write these problems on the board and distribute the story-workbooks. Let the students work independently for a while.

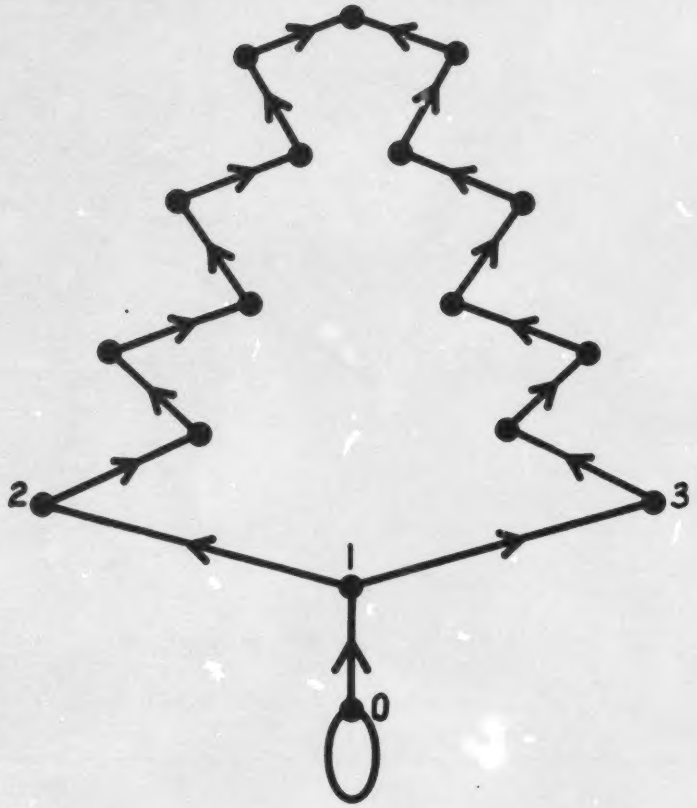
**	Build a road between 0 and 100.
***	Solve the problem on page 29.
****	Solve the problem on page 27.

As the students finish this work, ask them to read and complete pages 31 through 39.

Before the end of the class, allow about five minutes to discuss the pictures on pages 37 and 39. A discussion might go like this.

Draw the picture from page 37 on the board.

T: Zero drew the arrows to show the direction going away from its house.



T: What do you think of this tree?

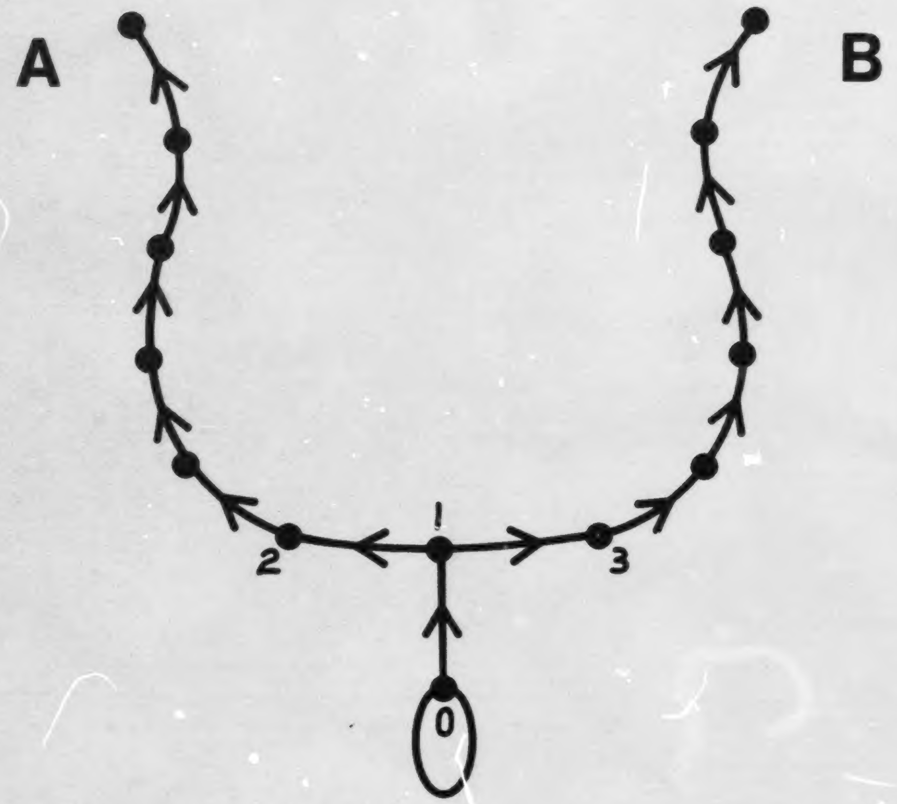
S: The top dot has two arrows ending there.

S: That can't happen. Earlier, we found that each dot had exactly one arrow ending at it and exactly two arrows starting at it.

T: Very good. What do you think of Zero's dream?

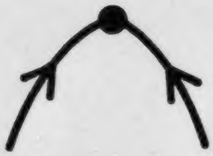
Draw the picture from page 39 and add arrows as shown in the next illustration.

T: In this picture the arrows show the way Zero walked and the way his friend walked.



T: Will the friends ever meet?

S: No, if they were to meet there would be a dot with two arrows ending at it. . .



. . . but each dot has only one arrow ending at it.

A Strange Country

THE RULE

The houses of two numbers are directly connected by a path

if and only if

ONE NUMBER EQUALS THE DOUBLE OF THE OTHER

or

ONE NUMBER EQUALS ONE MORE THAN THE DOUBLE OF THE OTHER

Are these houses directly connected?

12 and 25 YES

7 and 5 NO

17 and 8 YES

28 and 14 YES

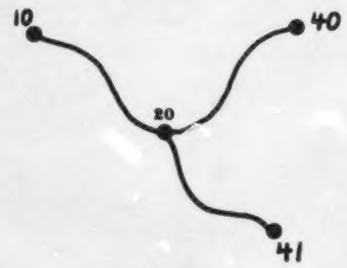
3 and 8 NO

27 and 13 YES

Come along. Let's go for a walk in the whole numbers' country.

● 20

Suppose we start at 20's house.
How many paths can we choose?
Who are the next neighbors of 20? Write their names in this picture.



Some days after the exhibition, 0 found on his desk the picture shown on page 37.
"It's strange," muttered 0.
By looking at the posters hanging on the walls of his office, he scribbled some calculations on a sheet of paper.
Finally, he wrote on the picture.

You are a little JOKESTER!

CAN YOU EXPLAIN WHY?

An explanation and discussion of this problem is given at the conclusion of Lesson 3.

During one night, 0 dreamed that he was going for a walk with one of his friends.
In front of 1's house, they had an argument. His friend decided to walk on road A while 0 chose road B. 0 awoke very upset because he realized that he and his friend would NEVER meet.

WHAT DO YOU THINK?

An explanation and discussion of this problem is given at the conclusion of Lesson 3.